LOGIC IN POLAND ATFER 1945 (UNTIL 1975)

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1. Introduction

This paper is conceived as a summary of logical investigations in Poland after 1945.[1] The date *ad quem* is determined by the death of Andrzej Mostowski, doubtless the most important Polish logician in after World War II (WWII for brevity) period. Times after this date are still too fresh in order to be subjected to a historical analysis. Thus, to finish at the moment of Mostowski's death does not only credit the memory of this great person, but well fits standards of historians related to understanding of what is the recent history; section 3 emphasizes Mostowski's role after 1945. Note, however, that when we use the past tense, it also applies to the present period in most cases. We concentrate on mathematical logic and the foundations of mathematics. Thus, works in semantics (except, logical model theory) and methodology of science, that is, other branches of general logic (logic *sensu largo*) are entirely omitted. Note, however, that scholars such as: Kazimierz Ajdukiewicz (Poznań, later Warszawa), Tadeusz Czeżowski (Toruń),Tadeusz Kotarbiński (Warszawa and Łodź) and Maria Kokoszyńska-Lutman (Wrocław), rather being logicians-philosophers than logicians-mathematicians, played an important role in teaching (in particular, they wrote popular

1 See also R. Wójcicki, J. Zygmunt, „Polish Logic in the Postwar Period", in *50 Years of Studia Logica*, ed. by V. F. Hendricks and J. Malinowski, Kluwer Academic Publishers, Dodrecht 2003, 11–33. Since this paper provides several data about names of active logicians after 1945 and the organization of departments of logic in the post-war Poland, we considerably restrict information about these issues.

textbooks of logic for a wide audience, including secondary schools), organizing Polish logical community after 1945 and undertaking several problems in the philosophy of logic (the nature of deduction, philosophical foundations of deductive systems, nominalistic foundations of logic). We also skip a description of works in the history of logic, one of favorites of Polish logicians, except mentioning reconstructions of some earlier systems. Another restriction consists of taking into account works of logicians living in Poland, not abroad. It causes some problems, because some people emigrated from Poland, but we hope that some liberty in operating the phrase "a logician living in Poland after 1945" is admissible. By mathematical logic we refer here to logical calculi and their metalogical properties, and the foundations of mathematics including set theory, general metamathematics, recursion theory and results about particular mathematical theories related to their various metamathematical properties. We are fully conscious that borderlines between mathematical logic and the foundations of mathematics and within this second field frequently overlap, but we think that this fact does not cause any fundamental difficulty for a historian of logic. Since our survey is concise, we try to outline the most important directions of research and results, although we are conscious that we follow our subjective views to some extent. We also omit bibliographical references in many cases (section 3 is an exception in this respect). Yet we hope that the above survey, although fragmentary to say once again, gives a general picture of how Polish logicians worked and what they achieved in mathematical (or formal) logic *sensu stricto*.

Before we go to our report, we would like to make few historical and sociological remarks. WWII had disastrous consequences (see also section 3) for logic in Poland (we will occasionally use the label 'Polish logic' as equivalent with 'logic in Poland'). Stanisław Leśniewski died before the war began, Stefan Banach in 1945. Władysław Hetper, Jan Herzberg, Adolf Lindenbaum, Moses Presburger, Józef Pepis, Jan Skarżeński and Mordechaj Wajsberg were murdered by the Nazis or perished in Soviet lagers (probably, Hetper, Herzberg, and Skarżeński). Leon Chwistek died in Moscow in 1944. Jan Łukasiewicz, Alfred Tarski, Henryk Hiż, Jan Kalicki, Czesław Lejewski and Bolesław Sobociński left Poland during 1939 –1948. In fact, Warsaw Logical School, the most powerful logic group in the interwar period, lost most of its representatives (Leśniewski, Lindenbaum, Łukasiewicz, Presburger, Sobociński, Tarski, Wajsberg); Stanisław Jaśkowski, Mostowski and Jerzy Słupecki became the only members of this community who remained in Poland after 1945. The losses of Polish logic (and the entire Polish science) during the war were not confined to

death of several scholars. They meant stopping normal research, teaching and international contacts for six years as well as destroying of public and private libraries. Many ready for publication or already prepared works disappeared. Yet, the clandestine universities trained some future important logicians, among others: Kalicki, Andrzej Grzegorczyk and Helena Rasiowa (the latter two graduated after 1945). The changes of Polish territory after 1945 resulted in losing two universities (Lvov and Vilna). In particular, the loss of Lvov, the second, after Warsaw, stronghold of Polish Mathematical School, was very significant. Although new universities were established (in Lublin, Łódź, Toruń and Wrocław, eventually also Katowice and Gdansk), it took time to organize normal work within them. Perhaps the University of Wrocław was in a relatively good situation, because its staff recruited from Lvov.

The post-war Poland inherited a very massive teaching of logic from the earlier period. Logic was taught at almost all university studies and in some pedagogical colleges. It was also present in secondary schools until the middle 1960s. The level of teaching varied in its character from very intensive at mathematics and philosophy to moderate at other faculties and in secondary schools. On the other hand, rudiments of mathematical logic were covered even by moderate curriculum. Teaching was realized by departments of logic existing in universities and other academic units. Typically, every university had at least one department of logic located at philosophical or mathematical faculty (the names of faculties were usually more complex, for instance, `philosophical-historical faculty' or `mathematical and physical faculty'). Some universities, for instance, in Warsaw and Wrocław had also departments of the foundations of mathematics. The special department of logic was organized in the Institute of Philosophy and Sociology of the Polish Academy of Science, and the group working in the foundations of mathematics was always very active within the Institute of Mathematics of the Polish Academy of Sciences Several good textbooks were published in the period in question. Let us mention a few examples (we mention books published by all-Polish publishers only): A. Mostowski, *Logika matematyczna* (Mathematical Logic, 1948), K. Kuratowski, A. Mostowski, *Teoria mnogości* (Set Theory, 1952 and subsequent extended editions; English edition, North-Holland, Amsterdam 1967), A. Grzegorczyk, *Zarys logiki matematycznej* (An Outline of Mathematical Logic, 1963 and several subsequent edition; English edition, Reidel, Dordrecht 1974) and L. Borkowski, J. Słupecki, *Elementy logiki matematycznej i teorii mnogości*, 1962 and several subsequent editions; English edition, Pergamon Press, Oxford 1967). The 1st volume of an international journal *Studia Logica* appeared in 1953; this journal

was organized by Ajdukiewicz. Another journals *Reports on Mathematical Logic* and *Bulletin of the Section of Logic* (The Institute of Philosophy and Sociology, Polish Philosophy and Sociology) was established in 1973. *Fundamenta Mathematicae* welcomed works in the foundations of mathematics as it continued a tradition of the prewar period. Moreover, particular universities and some pedagogical colleges had (and have) own journals in which logicians could publish papers. Poland hosted several important symposia and conferences in logic and the foundations. Let us mention a famous symposium "Infinistic Methods" (Warszawa 1959), the Logic Semester at the Banach Center (Warszawa 1973) and the Logic Colloquium (Karpacz 1975). Mostowski's seminar at the University of Warsaw became a very important place of training logicians coming from all parts of Poland and the world.

As it should be expected the life of logic in Poland was also related to the general political climate under communism. Basically, mathematical logic and the foundations of mathematics were considered ideologically neutral in Poland and most other communist countries, including Soviet Union. On the other hand, clear links of these fields to general philosophy made the situation, perhaps not very difficult, but still delicate. The official domination of Marxism in philosophy and ideology influenced the style of thinking inside the communist part of the world after 1945. The tension between authentic scientific standards and requirements of ideology was usually strongly dependent on the actual political situation and its pressure, but everybody had to take into account that he or she could be accused of idealism or other philosophical sins. This situation resulted in a separation of logic and philosophy. In the case of Poland it was particularly important, because strong co-operation of mathematics and philosophy was one of the most important circumstances determining the real power of Polish logical community. The new political situation created a new attitude: mathematical logicians felt more identified with mathematicians than philosophers. This remark should be somehow qualified. In fact, the separation of logic (particularly, the foundations of mathematics) and philosophy became a world-standard after 1945. Thus, this process could reach Poland independently of political circumstances. Since we abstain from counterfactual statements concerning what would happen, if …, we note the fact without entering into risky diagnoses that Polish logic could be exceptional as compared with other logical centers. Finally, let us indicate that one feature of the prewar tradition of logical investigations in Poland was fully continue after 1945. Polish logicians did not limit their research by great foundational schemes, like logicism, intuitionism or formalism. This means

that all methods were admitted, finitary or not. Perhaps this attitude contributed to defense of logic in Poland against ideological claims.

2. Mathematical logic

This part of our study concerns investigations of Polish logicians on logical calculi and their theory done after 1945. We will characterized general tendencies as well as report some concrete results (for bibliographical and substantial details see W. A. Pogorzelski, *Notions and Theories of Elementary Formal Logic*, Warsaw University – Bialystok Branch, Bialystok 1994 and R. Wójcicki, *Theory of Logical Calculi. Basic Theory of Consequence Operations*, Kluwer, Dodrecht 1988). To some extent, this research followed the tradition established by Łukasiewicz and his school before 1939 consisting in formulating logical systems and investigating their metalogical properties. Yet, due to numerous axiomatizations of logics elaborated by Łukasiewicz's and his students, metalogic played much more important role in Polish works done after World War II. In general two directions of research can be distinguished in metalogic. One, which can be termed as external, consisted in looking at logical systems from the point of view of mathematical methods used in investigations. The second direction, internal so to speak, employed methods remaining within logic itself or starting from their generalization at the metalogical level. Clearly, both directions cannot be entirely separated or contrasted and were frequently executed by the same persons. Roughly speaking, the external standpoint was characteristic for logicians working at mathematical faculties, but the internal approach was characteristic for researchers associated with philosophical faculties.

Helena Rasiowa's and Roman Sikorski's book *The Mathematics of Metamathematics* Państwowe Wydawnictwo Naukowe, Warszawa 1962 can be considered as perhaps the most representative summary of the external direction. This monumental work provides a general uniform algebraic and topological frame for several logics, (propositional and first-order) including classical logic, intuitionistic logic, positive logic and modal logics (more precisely, Lewis' modal systems). Exposition of logics as such is preceded by an account of formalized theories cum the algebra of formalized language. This approach leads to algebraization and topologization of syntax and semantics. The authors give the proof of completeness theorem for propositional and (first-order) predicate calculus by the Boolean method (both proofs were earlier published by the authors). Related proofs of the completeness theorem were given by Mostowski (1948; he translated the concept of satisfaction into topology) and Jerzy Łoś (1951, 1955; he used the ultraproduct construction; see section 3). Rasiowa and Sikorski

investigated non-classical (positive, intuitionistic and modal logics) from the classical perspective. Otherwise speaking, they freely applied classical logic in metamathematics and metalogic as it was required by the founding fathers of Polish School of Logic. This strategy allowed proving the completeness theorem for elementary intuitionistic logic. Rasiowa's book *An Algebraic Approach to Non-Classical Logics*, H. Rasiowa, *Algebraic Methods in Non-Classical Logics*, North-Holland, Amsterdam 1974 extended the external (mostly algebraic perspective) to a wide variety of non-classical logics, including (except earlier mentioned systems) many-valued logic and constructive logic with strong negation. Several students of Rasiowa, namely Andrzej Białynicki-Birula, Grażyna Mirkowska, Ewa Orłowska, Eleonora Perkowska, Cecylia Rauszer, Andrzej Salwicki, Andrzej Skowron, Jerzy Tiuryn and Anita Wasilewska continued this line of research; in particular. Certainly, the external point of view opened new and very important horizons for logical investigations. On the other hand, since it considered logic *sub species* mathematics, its actual significance for logic as such was (and still is, although mathematicians perhaps would protest) considerably limited.

The internal point of view had much more representatives in Poland after 1945. The following groups are to be mentioned (since we systematize by places and names associated with them, some names appear more than once; please also remember that 1975 is the deadline; we are sorry, if some persons are omitted): Warszawa (Roman Suszko, Henryk Greniewski, Ryszard Wójcicki, Jerzy Łoś, Zdzisław Pawlak, Andrzej Grzegorczyk, Zbigniew Lis, Zdzisław Kraszewski, Mieczysław Omyła, Jacek Malinowski, Zdzisław Ziemba), Wrocław and Opole (Jerzy Słupecki, Ludwik Borkowski, Tadeusz Kubiński, Witold A. Pogorzelski, Juliusz Reichbach, Katarzyma Hałkowska, Krystyna Piróg-Rzepecka, Urszula Wybraniec-Skardowska, Grzegorz Bryll, Janusz Czelakowski, Tadeusz Prucnal, Paweł Bielak, Jacek Hawranek, Bolesław Iwanuś, Marian Maduch, Zbigniew Stachniak, Jan Zygmunt), Kraków (Kazimierz Pasenkiewicz, Stanisław J. Surma, Andrzej Wroński, Jerzy Perzanowski, Jacek Kabziński, Jan Zygmunt, Piotr Krzystek, Barbara Woźniakowska, Małgorzata Porębska, Ewa Capińska), Poznań (Seweryna Łuszczewska-Roman, Tadeusz Batóg, Wojciech Buszkowski, Kazimierz Świrydowicz), Katowice (Witold A. Pogorzelski, Piotr Wojtylak, Wojciech Dzik, Marek Tokarz), Lublin (Ludwik Borkowski, Jacek Paśniczek), Łódź (Grzegorz Malinowski), Toruń (Stanisław Jaśkowski, Leon Gumański, Jerzy Kotas, August Pieczkowski, Wojciech Dziobiak, Zbigniew Rogowski) and Gdańsk (Zbigniew Rogowski).

Wójcicki's *Theory of Logical Calculi. Basic Theory of Consequence Operations* and Pogorzelski's *Notions and Theories of Elementary Formal Logic* (both mentioned above) are

comprehensive summaries of the internal approach to logic. The subtitle “Basic Theory of Consequence Operations” indicates one of the main points of this line of research. Logical matrices became second device for investigating logical calculi in this tradition. Roughly speaking, it follows famous paper by Łukasiewicz and Tarski, “Investigations about Sentential Calculus” published in 1929 and Tarski’s works on consequence operation published in the years 1930–1939. However, although the notion of consequence operation finds its application in any logic, the use of logical matrices is limited to propositional calculus. Yet the semantic treatment via matrices (the matrix semantics) provides a uniform general frame for all propositional logics, including classical, many-valued (finite and infinite), intuitionistic or intermediate (between classical system and intuitionistic system). Relating the concept of consequence operation and the concept of logical matrix (and of the concept of model in the case of predicate calculus) allowed to refine many important logical notions as the size of a logical matrix, distinguish various kinds of consequence operation, types of rules of inference (for instance, admissible, structural, derivable, substitutional, etc.) and more closely connect natural deduction (as invented by Jaśkowski in the 1930s) with semantics. It is no exaggeration to say that Polish works on the propositional logic constituted the most complete theory of this sub-domain of logic. Clearly, it was continuation of Łukasiewicz’s tradition as focusing on propositional calculus as a laboratory for logical research.

The following more detailed investigations are worthy to mention: the semantics for intuitionistic logic (Grzegorczyk; he anticipated Kripke’s construction). Non-Fregean logic (Suszko; this logic introduces the identity connective and contrasts it with the equivalence functor; this distinction forms a basis for a situation semantics for propositional logic), disscussive and paraconsistent logic (Jaśkowski), the rejection consequence operation (Słupecki, Bryll, Wybraniec-Skardowska; formalizes the concept of rejection), the dual consequence operation (Wójcicki; another approach to the concept of rejection), rough logic (Pawlak, a kind of fuzzy logic), intermediate logics (Wroński; logics between intuitionistic system and classical system), the deduction theorem and the Lindenbaum maximalization lemma (Surma, Pogorzelski; detailed studies on the scope of both theorems), general theory of completeness (Pogorzelski-Wojtylak), the Łoś-Suszko theorem on matrix consequences), applications of logic to theoretical linguistics (Buszkowski, Tokarz; in particular, studies on logical foundations of categorical grammar and pragmatics), completion of Łukasiewicz’s analysis and axiomatization of Aristotelian logic (Słupecki), studies on systems of many-valued logic (see G. Malinowski, *Many-Valued Logics*, Oxford University Press, Oxford

1993, R. Wójcicki, G. Malinowski (eds.), *Selected Papers on Łukasiewicz Sentential Calculi*, Ossolineum, Wrocław 1977) and analysis and extension of Leśniewski's systems (Słupecki, Iwanuś, Stachniak, Grzegorczyk). The following English editions of works of older scholars appeared: S. Leśniewski, *Collected Works*, Kluwer Academic Publishers, Dodrecht 1992, J. Łukasiewicz, *Elements of Mathematical Logic*, Pergamon Press, Oxford 1963, J. Łukasiewicz, *Selected Works*, North-Holland 1970, A. Mostowski, *Foundational Studies* 2 vols., North-Holland, Amsterdam 1979, M. Wajsberg, *Logical Works*, Ossolineum, Wrocław 1977; collections of papers by Łukasiewicz and Tarski and their books (Łukasiewicz, *Elements of Mathematical Logic*, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, Tarski, *Introduction to Logic and to the Methodology of Deductive* Sciences) appeared in Polish as well.

3. The Foundations of Mathematics

In this part of our article we will discuss the most important contributions made to foundations of mathematics in Poland during the period 1945 through 1975. This period started with the end of WWII, and ended with the death of Andrzej Mostowski, who dominated the foundational research during that time. One should not conclude that the research in mathematical logic and, more generally, Foundations, was limited to the Mostowski's Circle. In fact some of the most important contributions were made in different centers. To put the work of the generation that had to reconstruct and organize research after WWII, one needs to recognize the geopolitical situation of Poland. Effectively, Poland was moved some 200 miles to the west ceding significant territory to the Soviet Union and taking over part of Germany. From the point of view of the purpose of this article, the significant event was the loss of Lvov, a Polish town in the east with the significant research in Foundations. Some elements of scientific infrastructure of Lvov were moved to Wrocław (previously German Breslau).

The other most important center of foundational investigations, Warszawa was over 90% destroyed including the University buildings. Even worse was the situation in the personal matters. Many important logicians (both on the mathematical side as well as philosophical side) and mathematicians interested in the foundations of mathematics, to repeat some data mentioned in section 1, died in the War (often as victims of the Holocaust or Stalinist repressions) or emigrated. These included as mentioned above, Stanisław Leśniewski (died in 1939), Stefan Banach who died in 1945, Adolf Lindenbaum (died in unknown circumstances), Moses Presburger (also died in unknown circumstances), Jan Łukasiewicz

(left Poland in 1944) and, perhaps most significantly, Alfred Tarski (left Poland in 1939). The limited communication with abroad, especially during the first 10 years of the period, contributed to a feeling of isolation of Polish scientists. On the other hand, that same period was characterized by significant scientific achievements of Andrzej Mostowski who became a mature scientist right before the beginning of WWII, survived German occupation, Warsaw Uprising of 1944, and the general disruption of scientific work during WWII and immediately afterwards. He received his habilitation (written during WWII) right after the end of WWII and after a short period of work at the Jagiellonian University came back to Warsaw. He was associated with Warsaw University and the Mathematical Institute of the Academy of Science in Warsaw till his death in 1975.

Mostowski built a major scientific center in the foundations in Warsaw. While there were some mathematicians of older generation (Sierpiński, Kazimierz Kuratowski to some extent also Stanisław Mazur) who were interested in foundational research, mostly in very limited way; Sierpiński working on combinatorial set theory, Kuratowski studied set-theoretical topology, Mazur studied computable real numbers theory, termed as computable analysis), there was at the beginning of the discussed period nobody who could help Mostowski to rebuild foundational studies. This situation changed later; with the help of Andrzej Grzegorczyk and Helena Rasiowa (and independently Wanda Szmielew in her work on various foundational topics, first decidability of some algebraic theories, and then on the foundations of geometry) Warsaw reappeared on the map of the world foundational investigations. Mostowski succeeded in keeping in touch with the research conducted by Tarski and others in the United States and Europe. In 1948 Mostowski went to the Institute of Advanced Study in Princeton where he renewed his contacts with Gödel (Mostowski visited Gödel in 1936 in Vienna and witnessed lectures of the latter on constructibility, thus consistency of the axiom of choice.) Throughout the period, Mostowski maintained his contacts with his Ph.D. advisor, Alfred Tarski. Given the reality of ``Cold War'' it was a risky venture, but Mostowski, with the assistance of the leadership of Polish mathematicians, succeeded in maintaining these contacts, to the extent possible.

The foundational research changed during the period of WWII, with new topics becoming involved with the new concepts and ideas. The new areas of investigations (beyond logical calculi and set theory) included recursion theory and model theory. The advent of recursion theory was grounded in the work of Gödel, Turing, Kleene, and others. To some extent the motivating areas included computability. Since the work of Turing and others it

became clear that computing machines will be constructed. There was an urgent need to understand what could be computed and how. The mathematical foundations of computing; what is now known as the computer science, had to be studied and various notions of computable functions, such as primitive recursive functions, recursive functions and other hierarchies had to be studied. Mostowski started this research during WWII (the documentation for his entire work of WWII has been destroyed when Warsaw was burned down in 1944). It was later reconstructed by Mostowski and published in his 'Definable sets of positive integers', *Fundamenta Mathematicae*, 34 (1947), pp. 81-112 (Mostowski's papers mentioned in this section are reprinted in A. Mostowski, *Foundational Studies*, quoted above). Specific results included the definition and fundamental properties of so-callled arithmetical hierarchy, that is hierarchy of first-order definable subsets of the set of non-negative integers. This research was grounded in the investigations of the projective hierarchy of sets of reals. Similarities and differences between these area drove one of the strands of Mostowski's research (together with A. Grzegorczyk and Cz. Ryll-Nardzewski, 'The classical and omega-complete arithmetic', *The Journal of Symbolic Logic* 23 (1958), pp. 188-206). A significant research effort of Mostowski was devoted to the incompleteness of mathematical theories containing arithmetic (the book by Tarski, Mostowski and Robinson mentioned above, and other papers of Mostowski) . That book provided an influential account of Gödel's work and its extensions. The work on hierarchies was continued by extending the investigations into transfinite; so-called Davis-Mostowski hierarchy of hyperarithmetic sets. In 1949, Mostowski studied algebraic technics for the investigations of intuitionistic logic ('The proofs of non-deducibility in intuitionstic functional calculus', *The Journal of Symbolic Logic* 13 (1948) pp. 204-207). This work continued and extended the work of McKinney and Tarski on algebraic investigations of logical calculi. Mostowski's work was further extended by Rasiowa (and then also by Sikorski) to provide algebraic methods for studies of semantics of various logical calculi (see also above). This work, further extended by a large research group in Warsaw later contributed to study of the foundations of computer science. The significant work in this area included so-called algorithmic logic (now known as dynamic logic) of Salwicki and Mirkowska.

Model theory, a new area of Foundations at the time, has been created by Tarski. Given the close relationship of Mostowski and Tarski, relationship that continued until Mostowski's death, it was only natural for Mostowski to work on model-theoretic research. We will later comment on the seminal contributions of Andrzej Ehrenfeucht, Jerzy Łoś and

Czeslaw Ryll-Nardzewski in this area. Mostowski established a number of fundamental results in the area of model theory, introducing several basic techniques. We will mention and explain two important contributions of Mostowski. The first one, on the dependence of the theory of the product of structures on the theories of factors (‘On direct products of theories’, *The Journal of Symbolic Logic* 17(1952) pp.1-31). These results, later expanded and generalized by Solomon Feferman and Robert Vaught belong to the basic techniques of the model theory. The second contribution, made by Mostowski jointly with his student Andrzej Ehrenfeucht introduced the technique of models with a given set of indiscernibles (‘Models of axiomatic theories admitting authomorphisms’, *Fundamenta Mathematicae* 43 (1956), pp. 50-68). This work matched nicely previous work of Mostowski (a technique used both in the doctoral dissertation and in his Habilitationsschrift) on models of set theory with urelements. As is common in every mathematical research, these results were subsequently generalized and extended by others. The point in us discussing both these contributions was in building a ``toolkit" that could (and was) used further by researchers of the area. Among many results obtained by Mostowski during the 1950ies one needs to refer to his joint work with Andrzej Grzegorczyk and Czesław Ryll-Nardzewski on mentioned above. During that period, Mostowski introduced generalized quantifiers (‘On a generalization of quantifiers’, *Fundamenta Mathematicae* 44(1957) pp. 12-36). This work, quite surprisingly related not only to investigations of so-called abstract logics but also to some fundamental concepts of Computer Science.

The 1960ies brought some change in Mostowski's interests; first, he contributed to no-standard logics (such as weak second-order logic). But the results of Paul J. Cohen on the independence of the continuum hypothesis and of the axiom of choice led to a significant change in Mostowski's interests. Originally slightly mysterious notion of forcing has been subject of intensive research all over the world, including Poland. Mostowski devoted a lot of attention to this area, culminating in a monograph devoted to the subject. Again, as was quite common in his work, Mostowski's interests evolved; in the 1970, his attention was shifting to so-called second-order theories such as second-order arithmetic and Kelley-Morse impredicative theory of classes (the research initiated already in 1950, see ‘Some impredicative definitions in the axiomatic set-theory’, *Fundamenta Mathematicae* 37 (1950), pp. 11-124). We omitted a number of other topics for which Mostowski is known: the first one is the Mostowski Collapse Lemma; every well founded set is isomorphic to a well-founded set with the membership relation (‘An undecidable arithmetic statement’, *Fundamenta*

Mathematicae 36 (1949) pp. 143-164). His studies of models of second order arithmetic preserving the notion of well-ordering (so-called beta-models) were an important topic of studies in a variety of areas, including reverse mathematics of Harvey Friedman. Mostowski was an author of several influential books (see section 1). Of these, besides of coauthored book with Tarski and Robinson mentioned above, the most important one is his book of 1965, namely A. Mostowski, *Thirty Years of Foundational Studies. Lectures on the Development of Mathematical Logic and the Study of the Foundations of Mathematics in* 1930–1964, Acta Philosophica Fennica, Helsinki 1965, that provided a unified perspective of the foundational research. His book (with Kuratowski) *Set Theory* (see bibliographical details in section 1) was for many years one of the basic books on the subject, particularly a textbook for many generations of students of set theory and more generally, mathematics.

Mostowski created a center of foundational research. His Warsaw seminar included (among others) Zofia Adamowicz, Wojciech Guzicki, Michal Jaegermann, Stanislaw Krajewski, Michal Krynicki, Andrzej Włodzimierz Mostowski, Roman Murawski, Janusz Onyszkiewicz, Marian Srebrny, Kazimierz Wiśniewski, P. Zbierski and one of the authors of this work. Long-time visitors from abroad included: Einar Fredriksson, Donato Giorgetta, Moshe Machover, Johann Makowsky and Jouko Vaananen, List of short-time visitors is too long to be stated.

The attention we devoted to the research of Andrzej Mostowski (understandable in view of the fact that one of the authors was his student) should not veil the fact that foundational research in Poland during the reported period resulted in many other significant achievements. Some of these were produced in Mostowski's circle of influence. Maybe the most important of it was research of Andrzej Ehrenfeucht who proved many important results during the initial period of studies of Model Theory, showing a variety of important theorems. We mentioned above the fundamental joint result with Mostowski on the models with indiscernibles. Other results of Ehrenfeucht included studies of topological methods in the theory of models and results on omitting types.

The work of Jerzy Łoś belongs to "heroic" period of Model Theory research. Of his many results (we mentioned his work with Grzegorczyk and Mostowski above) we will state three results that are present in every book on model theory. The first one (usually called Łoś-Tarski theorem) characterizes formulas preserved downwards. Specifically, given a theory T, formulas preserved "downwards" from infinite models of T to infinite substructures that are also models of T, are those that are equivalent in T to universal formulas. A result

characterizing elementary classes (i.e. collections of structures satisfying a first-order theory) closed under "increasing unions" as those that are models of an universal-existential theory is commonly known as Chang-Łoś-Suszko theorem. The most important technique introduced by Łoś is that of ultraproduct. It deals with the construction of new structures from anindexed family of structures (and an ultrafilter in the index-set). This technique is one of fundamental construction of model theory and is widely used (J. Łoś, 'Quelques remarques, théorémes et problèmes sur le classes définissables d'algèbres', in: *Mathematical Interpretation of Formal Systems*, pp. 98--113, North Holland, Amsterdam 1955).

Grzegorczyk's best known and important result (see also information about his work with Mostowski and Ryll-Nardzewski above) is the so-called Grzegorczyk hierarchy (*Some Classes of Recursive Functions,* Instytut Matematyczny PAN, Warszawa 1953). He described and investigated classes of recursive functions which can be obtain by applying superposition, restricted recursion and the operation of minimum from some prescribed basic functions containing addition, multiplication and, additionally, satisfying the condition that every class in question includes more complicated primitive recursive functions. The resulting subrecursive hierarchy fill the class of primitive recursive functions.

During the period 1945 --1975, besides of the work of Mostowski, Grzegorczyk, Rasiowa, Sikorski, Szmielew and their collaborators in Warsaw, a significant work on foundations of mathematics were done in Wrocław. The most significant work in this area was done by Czeslaw Ryll-Nardzewski and also by Hugo Steinhaus and Jan Mycielski. Ryll-Nardzewski proved that first-order Peano arithmetic is not finitizable; there is no finite axiomatization for this, one of most fundamental in mathematics, theory. After Ryll-Nardzewski argument, Mostowski obtained a different proof of the same result. Among other results of Ryll-Nardzewski, one needs to mention his criterion for categoricity of first-orded complete theories in the power omega. Specifically, Ryll-Nardzewski proved that such theory is categorical in power omega if and only if for all k, there is only finitely many k-types.The axiom of determinacy of Mycielski and Steinhaus asserts that the Banach-Mazur game of length omega is determined (i.e. one of players must possess a winning strategy). The statement of the axiom of determinacy is inconsistent with the axiom of choice but has many attractive consequences. Axiom of determinacy played an important role in further developments of set theory and was studied in many leading centers of foundational investigations.. Besides of Mycielski, Ryll-Nardzewski and Steinhaus, foundational research

in Wrocław was conducted by Leszek Pacholski, Jan Waszkiewicz, Bogdan Węglorz, Agnieszka Wojciechowska, Andrzej Zarach and others.

This short review should convey the message of vibrant research activities principally conducted in Warszawa and Wrocław, with the participation of scientists from other places. As in every human activity, the importance of human discoveries lies in inspiring others to extend the line of activities and to find competing and complementary images of the area of investigation. In short to provide “shoulders” on which the subsequent generations of researchers can stand. With this perspective, it should be clear that the foundational research in Poland during the reported period (1945-1975) played that role. Due to specific research areas (set theory, model theory) it provided an important both the significant meeting place for the global effort in foundations, and a number of techniques that found its permanent place in logic, both philosophical and mathematical. Given the difficult external conditions (“Cold War”) it is even more amazing that at that place and at that time so much would happen.