Theory of Computation **CS 575**
Spring 2016 **Practice Final Exam**

Note that this is longer than the actual exam.

For each problem, explain your answer or show how it was derived.

1. (a) Give a NFA that accepts the language generated by the regular expression $(a + ba)^*(b + ab)^*$.
   (b) Convert your NFA to a DFA.

2. For each of the following sets, determine which category it's in.

   - Regular
   - Context-free and not regular
   - Not context-free
   - In P (and not regular or context-free, if encoding details sufficiently specified to prove this)
   - In NP
   - $\leq_m^P$-complete for NP
   - Decidable
   - Recognizable, not decidable
   - Not recognizable

   For each language, prove your answer. You may assume that the following languages are $\leq_m^P$-complete for NP: SAT, 3SAT, Vertex Cover, Clique, Hamiltonian Cycle, Traveling Salesperson.

   (a) $\{a^p : p \text{ prime}\}$
   (b) UniqueSAT $= \{\varphi : \varphi$ has **exactly one** satisfying assignment$\}$
   (c) $\{w \in \{a, b\}^* : |w| > 3 \wedge w \text{ has an even number of } b\text{s}\}$
   (d) $\{T : |\mathcal{L}(T)| \text{ is finite and divisible by } 2\}$
   (e) $\{a^i b^j c^k : i \leq j \leq k\}$
   (f) Set Cover: Given a universe $U = \{1, \ldots, n\}$ and a set $S \subset \mathcal{P}(U)$ of subsets of U, and $k \in \mathbf{N}$, is there a set $S' \subseteq S$ of k subsets of U whose union is U?
   For instance, if $n = 4$ and $S = \{\{1, 2\}, \{1, 3\}, \{1, 4\}\}$, then $\langle U, S, 3\rangle$ is in Set Cover, but $\langle U, S, 2\rangle$ is not.

3. Consider this CF language:

$$\{a^n b^k a^n : n, k \in \mathrm{N}\}.$$

   (a) Give a Chomsky normal form grammar for the language (Suggestion: first give a non-CNF grammar, then convert it.)
   (b) Using one of those grammars (CNF or not), give a PDA for this language.

4. True or False? (Justify your answer.)

   (a) If L_2 is context-free, and $L_1 \subseteq L_2$ then L_1 is regular.
   (b) If L is not regular then L^* is not regular.
   (c) It is possible that the union of a regular language and a nonregular language is regular.