

1. What do we know about the deterministic time and space complexity of co-NP? (Give a brief argument that your claims are true.)

2. True or False:

- If  $L_1$  is not recognizable, and  $L_1 \subseteq L_2$ , then  $L_2$  is not recognizable.
- If  $L$  is NP-complete, and  $L \leq_m^P S$ , then  $S$  is NP-complete.
- If  $L$  is NP-complete, then  $L$  is decidable in exponential time.
- If  $L$  is decidable in exponential time, then  $L$  is NP-complete.

3. Reductions

- a) What does it mean that  $f$  is a reduction from  $A$  to  $B$ ?
- b) If we know  $A \leq^P \text{SAT}$ , what can we conclude about  $A$ ?
- c) If we know  $\text{SAT} \leq^P A$ , what can we conclude about  $A$ ?

For each of the following languages, choose one of the following classes, and prove that this language is in that class: **in P**, **in NP**, **decidable**, **semidecidable**, or **not semidecidable**. The best grade will be given for the tightest bound.

4. The set of graphs that have paths of length at least 3.

5. SET COVER: Given a universe  $U = \{1, \dots, n\}$  and a set  $S \subset \mathcal{P}(U)$  of subsets of  $U$ , and  $k \in \mathbf{N}$ , is there a set  $S' \subseteq S$  of  $k$  subsets of  $U$  whose union is  $U$ ?

For instance, if  $n = 4$  and  $S = \{\{1, 2\}, \{1, 3\}, \{1, 4\}\}$ , then  $\langle U, S, 3 \rangle$  is in Set Cover, but  $\langle U, S, 2 \rangle$  is not.

6.  $L = \{e(T) : T \text{ accepts no more than 3 distinct inputs}\}$ .