Discrete Math **CS 275**
Fall 2008 **First Exam Practice Problems**

For each problem, explain your answer or show how it was derived.

1. For each of the following relations and each property { symmetric, anti-symmetric, reflexive, transitive}, state whether it has that property.
   (a) $R_1(x, y)$ holds iff x is y's sister, where x and y are drawn from all living women.
   (b) $R_2(x, y)$ holds iff $x + y > 0$, where $x, y \in \mathbf{R}$
   (c) $R_3(x, y)$ holds iff there is a road with no intersections connecting x and y, and x, y are intersections in Lexington.
2. If $f : A \to B$ is onto and $g : B \to C$ is one-one, what can we say about the relative values of $|A|$, $|B|$, and $|C|$?
3. Prove or disprove that $[(x \Rightarrow y) \Rightarrow z] \Rightarrow (x \Rightarrow z)$.
4. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5\}$.
   (a) Give an element of $A \times B$
   (b) What is $|A \times B|$?
   (c) Give an element of $A \oplus B$
   (d) What is $|A \oplus B|$?
5. Suppose you are told that $|A| = 7$, $|B| = 12$, and $|A \cup B| = 15$. What is $|A \oplus B|$? (To insure partial credit if you are unsure of your answer, indicate how you derived the answer, showing a formula, a Venn diagram, or other.)
6. Consider the following functions. For each function and each property from the set {one-one, onto, total}, list the properties that that function has.
   (a) $f : \mathbf{N} \to \mathbf{R}, f(x) = \log_2(x)$
   (b) $g : \mathbf{N} \times \mathbf{N} \to \mathbf{N}, g(\langle x, y \rangle) = 2^x 3^y$
   (c) $h : \mathbf{R}^+ \to (0, 1), h(x) = \frac{1}{x+1}$.
7. For each of the pairs f and g, state whether f is $\mathcal{O}(g)$, g is $\mathcal{O}(f)$, or f is $\Theta(g)$.
   (a) $f(n) = n$ and $g(n) = n \log n$
   (b) $f(n) = n^2$ and $g(n) = n \log n$
   (c) $f(n) = 2^n$ and $g(n) = n!$

8. Let

$$S(n) = \sum_{\{a_1,a_2,\ldots a_k\}\subseteq\{1,2,\ldots,n\}} \frac{1}{a_1 \cdot a_2 \cdots a_k}.$$

Here, the sum is over all nonempty subsets of $\{1, 2, \ldots n\}$.

Note: $S(3) = (\frac{1}{1} + \frac{1}{2} + \frac{1}{1\cdot 2}) + (\frac{1}{3}) + (\frac{1}{3\cdot 1} + \frac{1}{3\cdot 2} + \frac{1}{3\cdot 1\cdot 2})$.

(a) What can you say about the third group of fractions, in terms of the first?

(b) How do you express $S(3)$ in terms of $S(2)$?

(c) Prove $S(n) = n$ by induction.