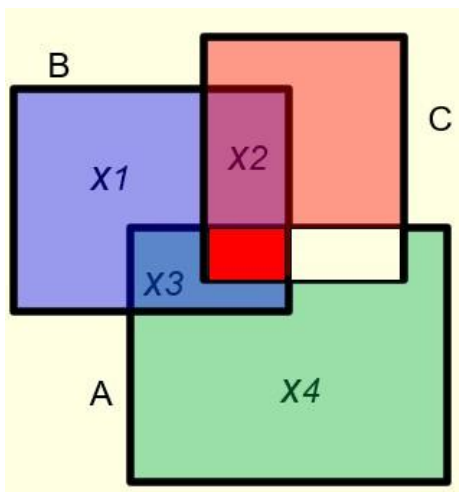


CS375 Homework Assignment 1 (40 points)

Due date: September 1, 2026

(The red boxes are text boxes. You can put your answers into the boxes directly)

1. In the following **Venn Diagram**, find an expression (in terms of set operations on the sets A, B and C; for instance, the green region can be expressed as $A - (B \cup C)$) for the region whose four components are marked with x_1 , x_2 , x_3 and x_4 , and put the expression in the red box below. The expression is not unique. Try to make your expression as compact as possible. (4 points)



Sol:

2. In the above diagram, if $|A| = 53$, $|C| = 50$, $|A \cap B| = 8$, $|A \cap C| = 10$, $|B \cap C| = 11$, $|A \cup B \cup C| = 145$ and $|A \cap B \cap C| = 6$ then $|B| = ?$ Show your derivation to get partial credit. (4 points)

Sol:

3. R is a binary relation over $\mathbb{N}$, the set of natural integers. $(x, y) \in R$ iff $x = y \pmod{5}$. R defined this way is an RST relation. List all equivalence classes of R in the following box. For each equivalence class, list all the elements in that class. (6 points)

Sol



4. Count the number of **strings** of length 5 over $A = \{a, b, c, d\}$ that begins with a , ends with b and have exactly one c . (5 points).

Sol:



5. Use the **pigeonhole principle** to determine how many people are needed in a group to say that at least **ten** were born on the same day of the week. (4 points)

Sol:

6. For a set A with m elements and a set B with n elements,

(i) how many different functions $f : A \rightarrow B$ can be defined if **m=4** and **n=5**? (3 points)

(ii) if **m=3** and **n=5** then how many different **one-to-one** functions $f : A \rightarrow B$ can be defined? (3 points)

(iii) if **m=4** and **n=3** then how many different **onto** functions $f : A \rightarrow B$ can be defined? (3 points)

7. Let $f: N_7 \rightarrow N_7$ be defined by $f(x) = (5x + 1) \bmod 7$. Find f^{-1} if it exists. Show your derivation to get partial credit. (4 points).

Sol:

8. Find a recursive definition for $f: N \rightarrow N$ defined by $f(n) = 0 + 1 + 8 + 27 + \dots + n^3$. (4 points)

Sol:

- Solutions must be typed (word processed) and submitted to Canvas both as a doc file and a pdf file before 23:59 on 09/01/2026.
- Name your files as:
 $CS375_HW1_2026f_Lastname.pdf$ / $CS375_HW1_2026f_Lastname.docx$