Homework 6: CS321, Fall 2017
Due Date: 3:15pm, December 5, 2017

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Show that the indefinite integral of a first-degree spline is a second-degree spline. (Hint: Let \( S(x) \) be a first-degree spline, define \( \overline{S}(x) = \int_0^x S(t)dt \). Show \( \overline{S}(x) \) is a quadratic spline by verifying all three conditions).

2. (10 points) Find a quadratic spline interpolant for these data

\[
\begin{array}{c|c|c|c|c|c}
 x & -1 & 0 & 1/2 & 1 & 2 \\
 y & 2 & 1 & 0 & 1 & 2 \\
\end{array}
\]

Assume that \( z_0 = 0 \).

3. (10 points) Determine if this function is a quadratic spline? Explain why or why not.

\[
Q(x) = \begin{cases} 
 x & -50 \leq x \leq 1 \\
 x^2 & 1 \leq x \leq 2 \\
 4 & 2 \leq x \leq 50 
\end{cases}
\]

4. (10 points) Do there exist \( a, b, c \) such that the function

\[
S(x) = \begin{cases} 
 ax^3 + x^2 + cx & x \in [-1,0] \\
 bx^3 + x^2 + dx & x \in [0,1] 
\end{cases}
\]

is a natural cubic spline function that agrees with the absolute value function \( |x| \) at the knots \(-1, 0, 1\)?

5. (10 points) Determine the coefficients \( a, b, c \) so that the function

\[
S(x) = \begin{cases} 
 x^3 & 0 \leq x \leq 1 \\
 \frac{1}{3}(x - 1)^3 + a(x - 1)^2 + b(x - 1) + c & 1 \leq x \leq 3 
\end{cases}
\]

is a cubic spline function.

6. (10 points) Determine the coefficients in the function

\[
S(x) = \begin{cases} 
 x^3 - 1 & -9 \leq x \leq 0 \\
 ax^3 + bx^2 + cx + d & 0 \leq x \leq 5 
\end{cases}
\]

such that it is a cubic spline function that takes the value 2 when \( x = 1 \).