Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Verify that the polynomials
   \[ p(x) = 3 + 2(x - 1) + 4(x - 1)(x + 2), \]
   \[ q(x) = 4x^2 + 6x - 7 \]
interpolate the data
   \[
   \begin{array}{c|ccc}
   x & 1 & -2 & 0 \\
   y & 3 & -3 & -7 \\
   \end{array}
   \]
and explain why this does not violate the uniqueness part of the theorem on existence of polynomial interpolation.

2. (10 points) Given the data
   \[
   \begin{array}{c|cccc}
   x & 1 & 2 & 2.5 & 3 & 4 \\
   y & -1 & -\frac{3}{4} & \frac{5}{12} & \frac{5}{3} & 25 \\
   \end{array}
   \]
Find an interpolating polynomial using Newton’s method.

3. (10 points) Find the polynomial \( p \) of least degree that takes these values: \( p(0) = 2, \)
   \( p(2) = 4, \)
   \( p(3) = -4, \)
   \( p(5) = 82. \)
   Use divide differences to get the correct polynomial.
   It is not necessary to write the polynomial in the standard form \( a_0 + a_1x + a_2x^2 + \ldots. \)

4. (10 points) Show that if a function \( g \) interpolates the function \( f \) at \( x_0, x_1, \ldots, x_{n-1} \)
   and \( h \) interpolates \( f \) at \( x_1, x_2, \ldots, x_n, \)
   then
   \[ g(x) + \frac{x_0 - x}{x_n - x_0}[g(x) - h(x)] \]
interpolates \( f \) at \( x_0, x_1, \ldots, x_n. \)

5. (10 points) Derive the approximation formula for the first derivative
   \[ f'(x) \approx \frac{1}{4h}[f(x + 3h) - f(x - h)] \]
and find its leading error term.

6. (10 points) If \( \phi(h) = L - c_1h^{1/2} - c_2h^{2/2} - c_3h^{3/2} - \ldots, \)
   then what combination of \( \phi(h) \) and \( \phi(h/2) \) should give an accurate estimate of \( L? \)