For each problem, explain your answer or show how it was derived.

1. For each of the following relations and each property \{ symmetric, anti-symmetric, reflexive, transitive \}, state whether it has that property.
   
   (a) \( R_1(x, y) \) holds iff \( x \) is \( y \)'s sister, where \( x \) and \( y \) are drawn from all living women.
   
   (b) \( R_2(x, y) \) holds iff \( x + y > 0 \), where \( x, y \in \mathbb{R} \)
   
   (c) \( R_3(x, y) \) holds iff there is a road with no intersections connecting \( x \) and \( y \), and \( x, y \) are intersections in Lexington.

2. If \( f: A \to B \) is onto and \( g: B \to C \) is one-one, what can we say about the relative values of \(|A|, |B|, \) and \(|C|\)?

3. Prove or disprove that \([(x \Rightarrow y) \Rightarrow z] \Rightarrow (x \Rightarrow z)\).

4. Let \( A = \{1, 2, 3, 4\} \) and \( B = \{1, 3, 5\} \).
   
   (a) Give an element of \( A \times B \)
   
   (b) What is \(|A \times B|\)?
   
   (c) Give an element of \( A \oplus B \)
   
   (d) What is \(|A \oplus B|\)?

5. Suppose you are told that \(|A| = 7, |B| = 12, \) and \(|A \cup B| = 15\). What is \(|A \oplus B|\)? (To insure partial credit if you are unsure of your answer, indicate how you derived the answer, showing a formula, a Venn diagram, or other.)

6. Consider the following functions. For each function and each property from the set \{one-one, onto, total\}, list the properties that that function has.
   
   (a) \( f: \mathbb{N} \to \mathbb{R}, f(x) = \log_2(x) \)
   
   (b) \( g: \mathbb{N} \times \mathbb{N} \to \mathbb{N}, g((x, y)) = 2^x3^y \)
   
   (c) \( h: \mathbb{R}^+ \to (0, 1), h(x) = \frac{1}{x+1} \).

7. For each of the pairs \( f \) and \( g \), state whether \( f \) is \( O(g) \), \( g \) is \( O(f) \), or \( f \) is \( \Theta(g) \).
   
   (a) \( f(n) = n \) and \( g(n) = n \log n \)
   
   (b) \( f(n) = n^2 \) and \( g(n) = n \log n \)
   
   (c) \( f(n) = 2^n \) and \( g(n) = n! \)
8. Let

\[ S(n) = \sum_{\{a_1, a_2, \ldots, a_k\} \subseteq \{1, 2, \ldots, n\}} \frac{1}{a_1 \cdot a_2 \cdots a_k}. \]

Here, the sum is over all nonempty subsets of \( \{1, 2, \ldots, n\} \).

Note: \( S(3) = \left( \frac{1}{1} + \frac{1}{2} + \frac{1}{1\cdot2} \right) + \left( \frac{1}{3} \right) + \left( \frac{1}{3\cdot1} + \frac{1}{3\cdot2} + \frac{1}{3\cdot1\cdot2} \right) \).

(a) What can you say about the third group of fractions, in terms of the first?

(b) How do you express \( S(3) \) in terms of \( S(2) \)?

(c) Prove \( S(n) = n \) by induction.