Preferences and Manipulative Actions in Elections

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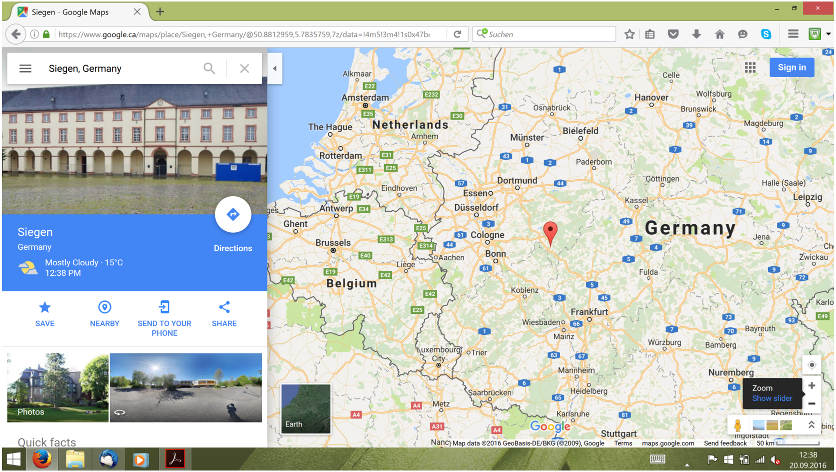
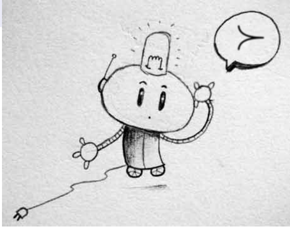
University of Kentucky

Where is Siegen?!



Preferences and Manipulative Actions in Elections



Outline

Computational Social Choice (COMSOC)

- Young, interdisciplinary area:
  - social choice theory,
  - computer science.

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- Young, interdisciplinary area:
  - social choice theory,
  - computer science.
- Research groups in:
  - law,
  - economics,
  - discrete mathematics,
  - decision theory,
  - theoretical computer science, and
  - artificial intelligence.

Merits of COMSOC

Contributes to both social choice and computer science by mutually transfering concepts between them.

- SOC $\rightarrow$ CS: Preference aggregation and collective decision making, e.g.,
  - Multi-agent planning,
  - Recommender systems,
- CS $\rightarrow$ SOC: Efficient algorithms, complexity of problems related to voting.

Main COMSOC Areas

- Voting theory
- Preference aggregation
- Resource allocation and fair division
- Coalition formation
- Judgment aggregation and belief merging

Where do we need preferences?

- Websearch,
- finding the best solution,
- finding best appointments,
- political elections,
- system configurations,
- multiagent planning,
- and many more...

Preferences can be very complex

Where should we go for lunch?

- Good mexican fast food place two bus stops away.
- Nice pizza for take-out one bus stop away.
- Sandwich in the fridge from yesterday, no loss of time.
- Juicy steak in the steakhouse around the corner, 5 min. walk.

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What is the best decision?

We have to consider: food, ambiance, distance, time, fare.

$\rightarrow$ Preferences can be very complex!

Voting Today

- Preference aggregation and collective decision-making.
- Political science, economics, social choice theory, and operations research.
- In computer science:
  - artificial intelligence (multiagent systems),
  - planning,
  - similarity search, and
  - design of ranking algorithms.

Elections

- Set of candidates and multiset of voters:
  - $C = \{c_1, \ldots, c_m\}$,
  - $V = \{v_1, \ldots, v_n\}$.
- Voter preferences over C can be represented as
  - preference lists (rankings),
  - approval/disapproval vectors,
  - CP-Nets (see, e.g., [Boutilier et al., 2004]).

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Example

Candidates: $C = \{$ THE SIMPSONS, the BIG BANG theory, THE IT CROWD $\}$ Voters: $V = \{$  $\}$

Profile $\mathcal{P}$:

Voting Rules

- Assume we know the agents' preferences
- . . . how can we come to a result?

Voting Rules

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- . . . how can we come to a result?
- We need a voting rule!
- A voting rule aggregates the preferences and outputs the set of winners:
  - unique-winner model,
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Voting Rules

- Assume we know the agents' preferences
- ... how can we come to a result?
- We need a voting rule!
- A voting rule aggregates the preferences and outputs the set of winners:
  - unique-winner model,
  - nonunique-winner model.
- What kind of voting rules exist?

Many Different Voting Rules

Many Different Voting Rules

Plurality

Borda

Many Different Voting Rules

Plurality

Veto

Borda

k-Approval

Many Different Voting Rules

Condorcet

Plurality

Bucklin STV

Veto

Borda

Black

Copeland

Maximin

k-Approval

Many Different Voting Rules

Slater

Dodgson

Condorcet

Plurality

Bucklin

STV

Veto

Borda

Black

Kemeny

Copeland

Maximin

Young

k-Approval

Scoring Rules

Given

- an election $E = (C, V)$ with $|C| = m$ and
- a *scoring vector* $\alpha = (\alpha_1, \ldots, \alpha_m)$

such that

- $\alpha_j \in \mathbb{N}$ for $1 \leq j \leq m$,
- $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m$ and
- $\alpha_1 > \alpha_m$.

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- $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m$ and
- $\alpha_1 > \alpha_m$.

Score: For $c \in C$ let $score(c) = \sum_{i=1}^{n} \alpha_j$, such that $pos(c, \succ_i) = j$

Winner: $w \in C$, such that $score(w) = \max_{c' \in C} score(c')$

Plurality

Plurality: $\alpha = (1, 0, \ldots, 0)$

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Example

Winner?

Plurality

Plurality: $\alpha = (1, 0, \ldots, 0)$

Example

	:	THE SIMPSONS	$\succ$	the BIG BANG theory	$\succ$	THE IT CROWD
	:	THE IT CROWD	$\succ$	the BIG BANG theory	$\succ$	THE SIMPSONS
	:	THE IT CROWD	$\succ$	THE SIMPSONS	$\succ$	the BIG BANG theory
	:	the BIG BANG theory	$\succ$	THE SIMPSONS	$\succ$	THE IT CROWD

THE SIMPSONS	:	1
the BIG BANG theory	:	1
THE IT CROWD	:	2

Winner?

Plurality

Plurality: $\alpha = (1, 0, \ldots, 0)$

Example

	:	THE SIMPSONS	$\succ$	the BIG BANG theory	$\succ$	THE IT CROWD
	:	THE IT CROWD	$\succ$	the BIG BANG theory	$\succ$	THE SIMPSONS
	:	THE IT CROWD	$\succ$	THE SIMPSONS	$\succ$	the BIG BANG theory
	:	the BIG BANG theory	$\succ$	THE SIMPSONS	$\succ$	THE IT CROWD

THE SIMPSONS	:	1
the BIG BANG theory	:	1
THE IT CROWD	:	2

Winner: THE IT CROWD

Borda Count [Borda, 1784]

Borda: $\alpha = (m-1, m-2, \ldots, 0)$

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Example

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	:	THE IT CROWD	$\succ$	the Big Bang theory	$\succ$	The Simpsons
	:	THE IT CROWD	$\succ$	The Simpsons	$\succ$	the Big Bang theory
	:	the Big Bang theory	$\succ$	The Simpsons	$\succ$	THE IT CROWD

Winner?

Borda Count [Borda, 1784]

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Example

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	:	The IT Crowd	$\succ$	The Big Bang Theory	$\succ$	The Simpsons
	:	The IT Crowd	$\succ$	The Simpsons	$\succ$	The Big Bang Theory
	:	The Big Bang Theory	$\succ$	The Simpsons	$\succ$	The IT Crowd

The Simpsons	:	4
The Big Bang Theory	:	4
The IT Crowd	:	4

Winner?

Borda Count [Borda, 1784]

Borda: $\alpha = (m-1, m-2, \ldots, 0)$

Example

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 :	THE IT CROWD	$\succ$	The Simpsons	$\succ$	The Big Bang Theory
 :	The Big Bang Theory	$\succ$	The Simpsons	$\succ$	THE IT CROWD

The Simpsons	: 4
The Big Bang Theory	: 4
THE IT CROWD	: 4

Winner: The Simpsons The Big Bang Theory THE IT CROWD

Condorcet Winner [Marquis de Condorcet, 1784]

The Condorcet Winner beats every other candidate in a pairwise election.

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Example

Winner?

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Example

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	:	THE IT CROWD	$\succ$	THE SIMPSONS	$\succ$	THE BIG BANG theory

THE SIMPSONS	vs	THE IT CROWD	THE IT CROWD	$>$	THE SIMPSONS	(2:1)
THE BIG BANG theory	vs	THE IT CROWD	THE IT CROWD	$>$	THE BIG BANG theory	(2:1)
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Winner?

Condorcet Winner [Marquis de Condorcet, 1784]

The Condorcet Winner beats every other candidate in a pairwise election.

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	:	THE IT CROWD	$\succ$	THE SIMPSONS	$\succ$	the BIG BANG theory

THE SIMPSONS	vs	THE IT CROWD	THE IT CROWD	$>$	THE SIMPSONS	(2:1)
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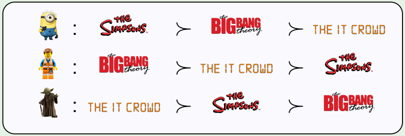
THE SIMPSONS	vs	the BIG BANG theory	THE SIMPSONS	$>$	the BIG BANG theory	(2:1)

Winner: THE IT CROWD

Condorcet Paradox

There are elections without a Condorcet winner!

Example



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The Simpsons	vs	THE IT CROWD	THE IT CROWD	$>$	The Simpsons	(2:1)
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Winner?

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	:	THE IT CROWD $\succ$ THE SIMPSONS $\succ$ THE BIG BANG THEORY

THE SIMPSONS vs THE IT CROWD	THE IT CROWD $>$ THE SIMPSONS (2:1)
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THE IT CROWD $>$ THE SIMPSONS

Winner?

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	: The Big Bang Theory	$\succ$	The IT Crowd	$\succ$	The Simpsons
	: The IT Crowd	$\succ$	The Simpsons	$\succ$	The Big Bang Theory

The Simpsons	vs	The IT Crowd	The IT Crowd	$>$	The Simpsons	(2:1)
The Big Bang Theory	vs	The IT Crowd	The Big Bang Theory	$>$	The IT Crowd	(2:1)
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The IT Crowd $>$ The Simpsons $>$ The Big Bang Theory

Winner?

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	:	The IT Crowd	$\succ$	The Simpsons	$\succ$	The Big Bang Theory

The Simpsons	vs	The IT Crowd	The IT Crowd	$>$	The Simpsons	(2:1)
The Big Bang Theory	vs	The IT Crowd	The Big Bang Theory	$>$	The IT Crowd	(2:1)
The Simpsons	vs	The Big Bang Theory	The Simpsons	$>$	The Big Bang Theory	(2:1)

The IT Crowd $>$ The Simpsons $>$ The Big Bang Theory $>$ The IT Crowd

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The Simpsons	vs	The IT Crowd	The IT Crowd	$>$	The Simpsons	(2:1)
The Big Bang Theory	vs	The IT Crowd	The Big Bang Theory	$>$	The IT Crowd	(2:1)
The Simpsons	vs	The Big Bang Theory	The Simpsons	$>$	The Big Bang Theory	(2:1)

The IT Crowd $>$ The Simpsons $>$ The Big Bang Theory $>$ The IT Crowd

Winner: **Nobody!**

Make the Right Choice!

Plurality: $\alpha = (1, 0, \ldots, 0)$

Borda: $\alpha = (m - 1, m - 2, \ldots, 0)$

Make the Right Choice!

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Example

	:	The Simpsons	$\succ$	The Big Bang Theory	$\succ$	The IT Crowd
	:	The Simpsons	$\succ$	The Big Bang Theory	$\succ$	The IT Crowd
	:	The Big Bang Theory	$\succ$	The IT Crowd	$\succ$	The Simpsons
	:	The IT Crowd	$\succ$	The Big Bang Theory	$\succ$	The Simpsons

Winner (Plurality) ?
Winner (Borda) ?

Make the Right Choice!

$$\text{Plurality: } \alpha = (1, 0, \ldots, 0)$$
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Example

	:	The Simpsons $\succ$ The Big Bang Theory $\succ$ The IT Crowd
	:	The Simpsons $\succ$ The Big Bang Theory $\succ$ The IT Crowd
	:	The Big Bang Theory $\succ$ The IT Crowd $\succ$ The Simpsons
	:	The IT Crowd $\succ$ The Big Bang Theory $\succ$ The Simpsons

	Plur.
The Simpsons :	2
The Big Bang Theory :	1
The IT Crowd :	1

Winner (Plurality) The Simpsons
Winner (Borda) ?

Make the Right Choice!

Plurality: $\alpha = (1, 0, \ldots, 0)$

Borda: $\alpha = (m-1, m-2, \ldots, 0)$

Example

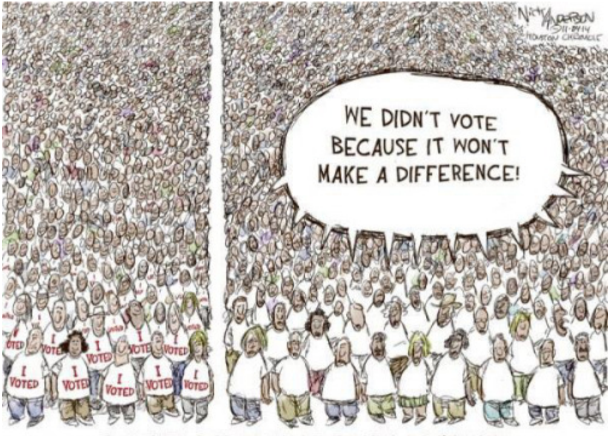
	: The Simpsons	$\succ$	The Big Bang Theory	$\succ$	The IT Crowd
	: The Simpsons	$\succ$	The Big Bang Theory	$\succ$	The IT Crowd
	: The Big Bang Theory	$\succ$	The IT Crowd	$\succ$	The Simpsons
	: The IT Crowd	$\succ$	The Big Bang Theory	$\succ$	The Simpsons

		Plur.	Borda
The Simpsons	:	2	4
The Big Bang Theory	:	1	5
The IT Crowd	:	1	3

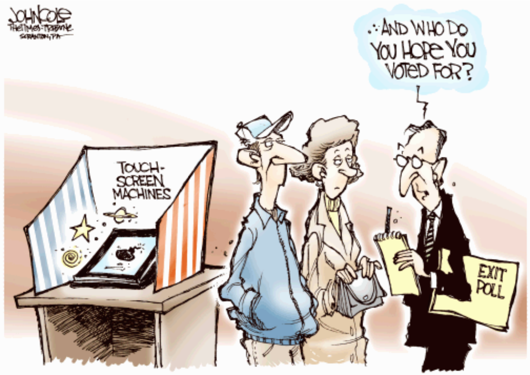
Winner (Plurality) The Simpsons

Winner (Borda) The Big Bang Theory

Properties



Manipulative Actions in Elections



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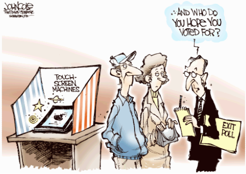
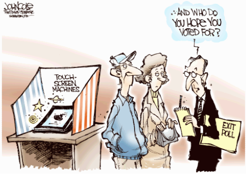


In real-world scenarios manipulative actions are possible!

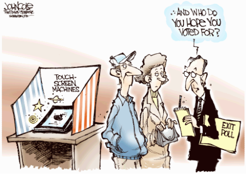
Manipulative Actions in Elections



In real-world scenarios manipulative actions are possible!

- **Manipulation**: An evil coalition of voters strategically change their votes.
- **Bribery**: An external agent bribes a group of voters.
- **Control:** The Chair modifies the election's structure.

Manipulative Actions in Elections



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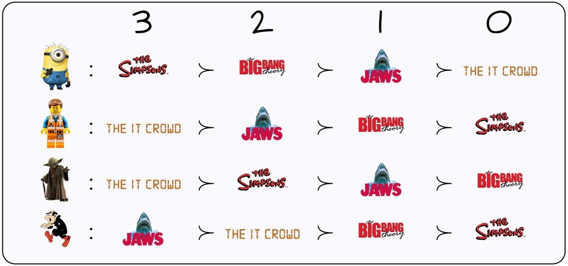
- **Control:** The Chair modifies the election's structure.

First papers on manipulation [Bartholdi et al., 1989], bribery [Faliszewski et al., 2006, Faliszewski et al., 2009] and control [Bartholdi et al., 1992]. For an overview we refer to the textbooks [Rothe, 2015, Brandt et al., 2016].

Manipulating Borda Count

		3		2		1		0
	:	THE SIMPSONS	$\succ$	the BIG BANG theory	$\succ$	JAWS	$\succ$	THE IT CROWD
	:	THE IT CROWD	$\succ$	JAWS	$\succ$	the BIG BANG theory	$\succ$	THE SIMPSONS
	:	THE IT CROWD	$\succ$	THE SIMPSONS	$\succ$	JAWS	$\succ$	the BIG BANG theory
	:	JAWS	$\succ$	THE IT CROWD	$\succ$	the BIG BANG theory	$\succ$	THE SIMPSONS

Manipulating Borda Count



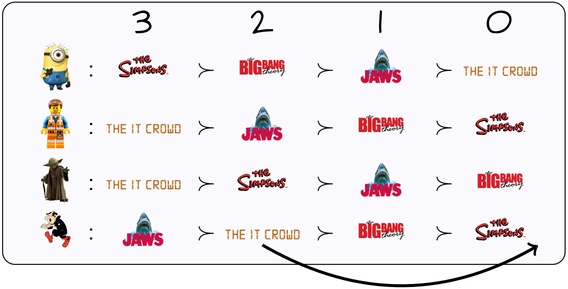
	THE SIMPSONS	BIG BANG theory	THE IT CROWD	JAWS
	3	2	0	1
	0	1	3	2
	2	0	3	1
	0	1	2	3
$\sum$	5	4	8	7

Manipulating Borda Count

		3		2		1		0
	:	The Simpsons	$\succ$	The Big Bang Theory	$\succ$	Jaws	$\succ$	The IT Crowd
	:	The IT Crowd	$\succ$	Jaws	$\succ$	The Big Bang Theory	$\succ$	The Simpsons
	:	The IT Crowd	$\succ$	The Simpsons	$\succ$	Jaws	$\succ$	The Big Bang Theory
	:	Jaws	$\succ$	The IT Crowd	$\succ$	The Big Bang Theory	$\succ$	The Simpsons

	The Simpsons	The Big Bang Theory	The IT Crowd	Jaws
	3	2	0	1
	0	1	3	2
	2	0	3	1
	0	1	2	3
$\sum$	5	4	**8**	7

Manipulating Borda Count



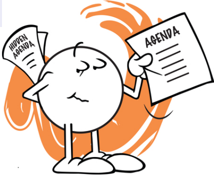
	THE SIMPSONS	BIG BANG theory	THE IT CROWD	JAWS
	3	2	0	1
	0	1	3	2
	2	0	3	1
	0	1	2	3
$\sum$	5	4	8	7

Manipulating Borda Count

		3		2		1		0
	:	THE SIMPSONS	$\succ$	the BIG BANG theory	$\succ$	JAWS	$\succ$	THE IT CROWD
	:	THE IT CROWD	$\succ$	JAWS	$\succ$	the BIG BANG theory	$\succ$	THE SIMPSONS
	:	THE IT CROWD	$\succ$	THE SIMPSONS	$\succ$	JAWS	$\succ$	the BIG BANG theory
	:	JAWS	$\succ$	the BIG BANG theory	$\succ$	THE SIMPSONS	$\succ$	THE IT CROWD

	THE SIMPSONS	the BIG BANG theory	THE IT CROWD	JAWS
	3	2	0	1
	0	1	3	2
	2	0	3	1
	1	2	0	3
$\sum$	6	5	6	**7**

Gibbard-Satterthwaite Theorem



Theorem (Gibbard 1973, Satterthwaite 1975)

Every strategy-proof voting system for three or more candidates must be dictatorial.

Complexity as Protection

- Immune (I): Manipulative action is impossible.
- Susceptible (S): Not immune.
- Vulnerable (V): S $+$ the corresponding problem is solvable in polynomial time.
- Resistant (R): S $+$ corresponding problem is computationally hard (i.e., NP-hard.)

Complexity as Protection

- Immune (I): Manipulative action is impossible.
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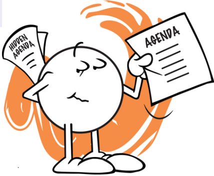
Worst-case complexity!

First Results Regarding Manipulation



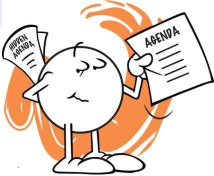
Theorem (Bartholdi, Tovey, and Trick 1989)

Single manipulation in

- *Copeland*$^\alpha$*,*
- *Maximin, and*
- *all scoring rules*

is solvable in polynomial-time.

Manipulation under STV



Theorem (Bartholdi and Orlin 1991)

STV-Manipulation *is* NP-*hard.*

Bribery

$\mathcal{E}$-Bribery

Instance: Election $E = (C, V)$, a nonnegative integer k, and a distinguished candidate $c \in C$.

Question: Is it possible to change at most k voters' votes such that c is a winner of the resulting election under voting rule $\mathcal{E}$?

Versions of Bribery

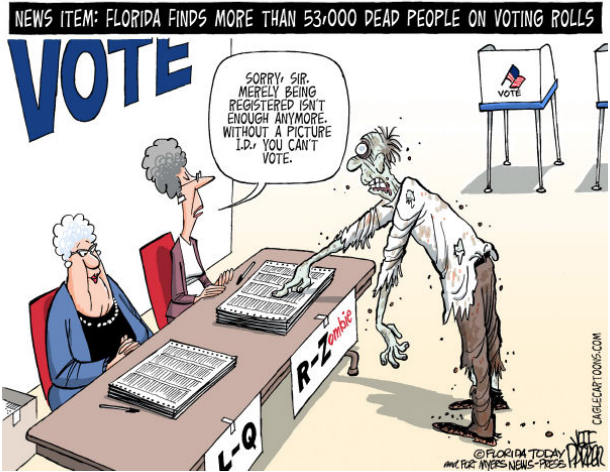
- $\mathcal{E}$-Bribery
- $\mathcal{E}$-\$Bribery: Voters have distinct prices, k is the limit.
- $\mathcal{E}$-Weighted-Bribery: Voters have weights.
- $\mathcal{E}$-Weighted-\$Bribery: Both weights and prices.
- $\mathcal{E}$-Microbribery: Each flip costs, k is the limit.

Electoral Control

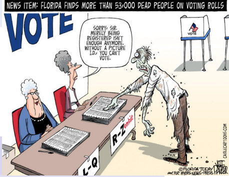
Basic Idea

The Chair seeks to influence the outcome of the election by changing the structure of it.

Electoral Control



Electoral Control



Basic Idea

The Chair seeks to influence the outcome of the election by changing the structure of it.

- Adding candidates (candidate recruitment),
- deleting candidates (forcing candidates out of race),
- partition of candidates without run-off (qualifying round for some candidates),
- partition of candidates with run-off (two groups of candidates, each voter votes on both groups separately),
- adding voters (get-out-the-vote drives),
- deleting voters (forcing voters out of the election), and
- partition of voters (two-district gerrymandering).

Contrast

Number of resistances, immunities, and vulnerabilities to the 22 common control types.

Number of	Copeland	Plurality	SP-AV	Bucklin	NRV	FV
resistances	15	16	19	$\geq$ 19	20	20
immunities	0	0	0	0	0	0
vulnerabilities	7	6	3	$\leq$ 3	2	2
References	[FHHR07]	[BTT92,HHR07]	[ENR09]	[EFRS15]	[Men13]	[EFRS15]

Is Our Model Right?

- full vs. partial information
- domain restrictions
    - single-peaked profiles
    - single-caved profiles
    - single-crossing profiles
    - top monotonicity
    - nearly single-peaked profiles

Domain Restrictions

Idea

Until now: Each admissible vote allowed

What if the diversity of votes is limited and the resulting profile offers some structure?

Domain Restrictions

Idea

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What if the diversity of votes is limited and the resulting profile offers some structure?

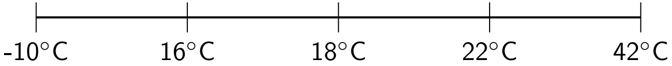
Advantages

- Better properties on restricted domains.
- Unnatural cases not present.
- Structure can help designing better algorithms.

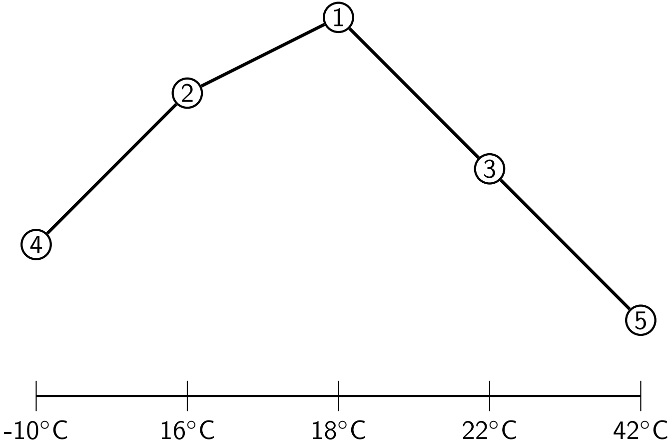
Single-Peakedness

Single-Peakedness

Single-Peakedness



Single-Peakedness



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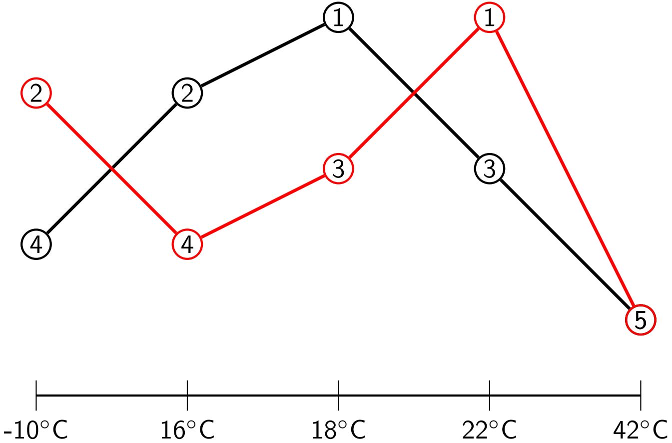

Single-peaked elections

Definition (Single-peakedness [Black, 1948])

Let an *axis* A be a total order on C denoted by $>$. Furthermore, let $\succ$ be a vote with top-ranked candidate c. The vote $\succ$ is *single-peaked with respect to* A if for any $x, y \in C$, if $x > y > c$ or $c > y > x$ then $c \succ y \succ x$ has to hold.

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A preference profile $\mathcal{P}$ is *single-peaked with respect to an axis* A if each vote is single-peaked with respect to A. A preference profile $\mathcal{P}$ is said to be *single-peaked consistent* if there exists an axis A such that $\mathcal{P}$ is single-peaked with respect to A.

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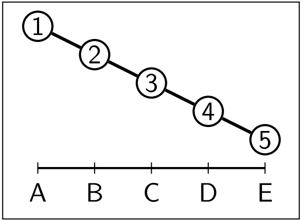
Can be decided (in $O(|C| \cdot |V|)$, vgl. [Escoffier et al., 2008])

Properties (amongst others):

- The Gibbard-Satterthwaite Theorem does not hold (see. [Moulin, 1980])
- Complexity often decreases (NP $\to$ P)

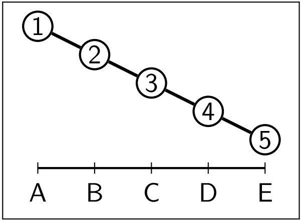
Problem: Single-peakedness is not robust!

Fragility of Single-Peakedness



100 votes

Fragility of Single-Peakedness



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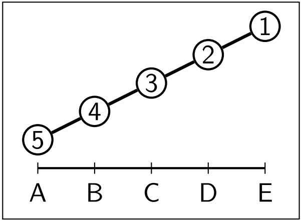

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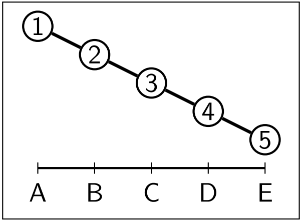
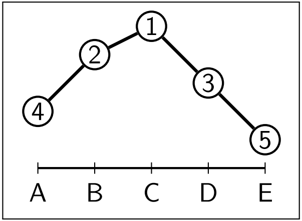
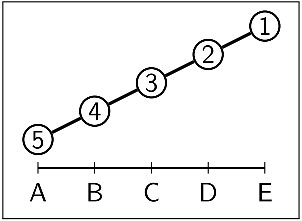


100 votes



100 votes



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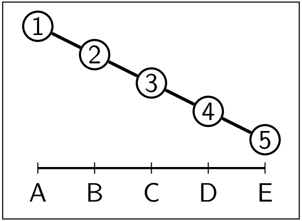

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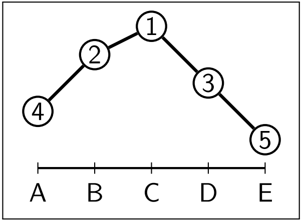

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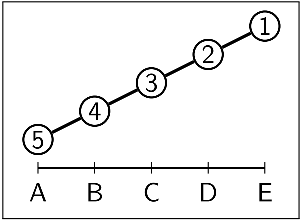

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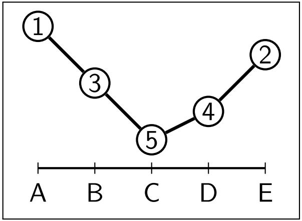

1 votes

Fragility of Single-Peakedness

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This is a big problem, since there is almost always some noise in preferences in real-world settings.

- Solution: There are robust versions of single-peakedness. However, complexity increases for single-peaked consistency. (See, e.g., [Faliszewski et al., 2014, Erdélyi et al., 2013, Bredereck et al., 2016])

Nearly single-peaked profiles

Define distance to single-peaked profiles.

Nearly single-peaked profiles

Define distance to single-peaked profiles.

1. k-Maverick Single-Peaked
2. k-Additional Axes Single-Peaked
3. k-Candidate Deletion Single-Peaked
4. k-Local Candidate Deletion Single-Peaked
5. k-Global Swaps Single-Peaked
6. k-Local Swaps Single-Peaked
7. k-Candidate Partition Single-Peaked

1. and 6. introduced by Faliszewski, Hemaspaandra, and Hemaspaandra [FHH11]
2. and 3. suggested by Escoffier, Lang, Öztürk [ELÖ08]

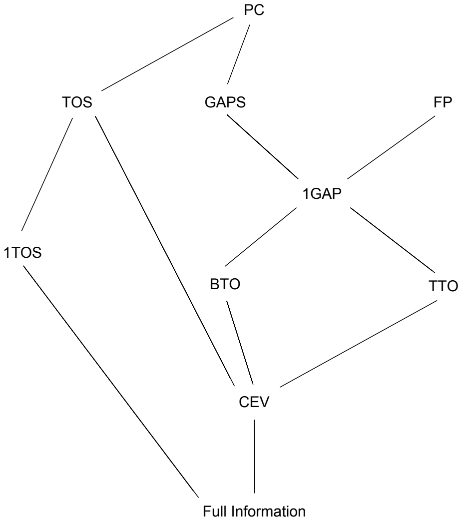
Complexity of Nearly Single-Peaked Consistency [ELP13]

k-Maverick	NP-c
k-Additional Axes	NP-c
k-Local Candidate Deletion	NP-c
k-Local Swaps	NP-c
k-Global Swaps	NP-c
k-Candidate Deletion	$\mathcal{O}(\lvert V\rvert \cdot \lvert C\rvert^5)$
k-Candidate Partition	?

Complexity of Veto-ℓ-X-CCWM [ELP15]

X	in P	NP-complete
Voter Deletion	$\ell \leq m-3$	$\ell > m-3$
Candidate Deletion	$\ell \leq m-3$	$\ell > m-3$
Local Candidate Del.	$\ell = 0$	$\ell \geq 1$
Global Swaps	$m = 2k$: $\ell \leq k^2 - k - 1$	$\ell > k^2 - k - 1$
	$m = 2k-1$: $\ell \leq k^2 - 2k$	$\ell > k^2 - 2k$
Local Swaps	$\ell < \lfloor \frac{m-1}{2} \rfloor$	$\ell \geq \lfloor \frac{m-1}{2} \rfloor$
Candidate Partition	$\ell < \frac{m}{2}$	$\ell \geq \frac{m}{2}$
Additional Axes	$\ell < \frac{m}{2} - 1$	$\ell \geq \frac{m}{2} - 1$

Incomplete Preferences [BER16]



Bribery Under Incomplete Preferences [BER16]

Voting rule	FI	GAPS	FP	TOS	PC	CEV	1TOS	1GAP	TTO	BTO
Plurality	P	**NPC**	**NPC**	**NPC**	**NPC**	**P**	**P**	**NPC**	**P**	**NPC**
2-Approval	P	**NPC**	**NPC**	**NPC**	**NPC**	**P**	**P**	**NPC**	**P**	**NPC**
($\geq$ 3)-Approval	NPC	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*
Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
($\geq$ 4)-Veto	NPC	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*

Dealing with NP-Hardness

Worst-case complexity vs.

- approximation algorithms
- algorithms that are always efficient although not always correct
- algorithms that are always correct, but not always efficient
- average-case complexity
- parameterized complexity

Conclusion

Some important points

- Elections have many real-world applications.
- Preferences and voting rules.
- Most voting rules susceptible to manipulative actions.
- Can computational complexity provide a shield?
- Natural restrictions.

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