

# CS 115 Lecture 20

## Recursion

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  - ▶ You can change the limit with `sys.setrecursionlimit(1000)`
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`RuntimeError("Maximum recursion depth exceeded...")`
  - ▶ So the program crashes before the OS does.
  - ▶ You can change the limit with `sys.setrecursionlimit(1000)`
    - ★ But then you risk crashing more than just your program!

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