

CS 115 Lecture 9

Boolean logic; random numbers

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Augmented assignment

Often you want to perform an operation on a variable and store the result in the same variable:

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num_students = num_students + 1
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price = price * 0.9 # 10 percent discount
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- Compares the numeric code (**Unicode**) for each character.
 - ▶ Mostly alphabetic for basic English characters.
 - ▶ Uppercase before lowercase! `'Z' < 'a'`
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The **truth table** is a tool for making sense of complex boolean expressions.

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- One row for each possible combination of values
 - ▶ If there is one input, two rows (T, F).
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- Let's look at one more: `random`

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