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Counterexample for Compactness

In the following I will construct a logic program P with infinitely many clauses with the property that for any finite subset F of P, there is a finite set F' of P which contains F such that F' has a stable model, but P has no stable model. Thus P is a counterexample for at least one version of a compactness theorem for stable models of logic programs.

P consists of the following clauses.

A)

$$a \leftarrow \neg a, \neg b$$

B)

$$b \leftarrow \neg c_i$$

for all $i \geq 0$.

C)

$$c_i \leftarrow$$

for all $i \geq 0$.

Note that P has no stable model. That is, if M is a stable model of P, then $c_i \in M$ for all i by the clauses in (C). Thus $b \notin M$ since the only way to derive b is from one of the clauses in (B) and these are all blocked for M. Now consider a. We cannot have $a \in M$ since the only way to derive a is via the clause in (A) but it would be blocked if $a \in M$. Thus $a \notin M$. However if $a \notin M$ and $b \notin M$, then we must have $a \in M$ by clause (A). This is a contradiction so that P has no stable model.

Now fix $n \geq 0$ and consider the following subprogram P_n of P.

P_n consists of the following clauses.

A)

$$a \leftarrow \neg a, \neg b$$

B)

$$b \leftarrow \neg c_i$$

for all $0 \leq i \leq n+1$.

C)

$$c_i \leftarrow$$

for all $0 \leq i \leq n$.

It is easy to see that P_n has exactly one stable model, namely $\{b, c_1, \ldots, c_n\}$. Moreover it is easy see that any finite subset F of P is contained in some P_n.