1. Prove that if $F_q \subseteq F_r \subseteq F_s$, then

$$\text{Tr}_q^r(\text{Tr}_r^s(x)) = \text{Tr}_q^s(x).$$

2. Let $f(x) = x^5 + x + 1$ be the connection polynomial of a LFSR over $F_2$.

   (a) If the LFSR starts in state 00001, what is the period of the output?
   (b) What does this tell you about the factorization of $f(x)$?
   (c) Use the Berlekamp-Massey algorithm to factor $f(x)$. Show your work.

3. How does the Berlekamp-Massey algorithm need to be modified so that it works over an arbitrary finite field $F_{p^d}$?

4. Let $G$ be the state graph of a feedback shift register (not necessarily linear), with feedback function $f(x_1, \ldots, x_n)$. Prove that $G$ decomposes as a disjoint union of cycles if and only if $f$ has the form

$$f(x_1, \ldots, x_n) = x_1 + h(x_2, \ldots, x_n)$$

for some function $h$.

5. Formulate a research question related to some topic we have covered this semester. It must be a question that you have not seen asked or answered anywhere. You do not need to answer the question.