Project 2: CS685-003, Spring 2002

Due Date: 1:50pm, March 1, 2002

This project is to write a parallel code using MPI and iterative methods for solving a one dimensional Poisson equation. You need to solve the equation $u_{xx} = f(x)$ defined on $\Omega = [0, 1]$. The Dirichlet boundary conditions and the right hand side function $f(x)$ are defined to satisfy the exact solution $u(x) = \sin(x)$. You will first need to discretize the equation using the second order central difference scheme with a uniform meshsize $h = 1/(n+1)$, where n is the number of interior grid points. Then you will use Jacobi and red-black (two color) Gauss-Seidel iterative methods to solve the sparse linear systems. The parallelization strategies will be implemented using *Message-Passing Interface*.

Here are the details of the programming project. **Your results should always be accompanied by comments**.

1. Given $n-2$ interior grid points and the number of processors p, each processor will have $(n-2)/p$ grid points. Note that input and output can only be performed by processor 0. The size of the subproblems $(n-2)/p$ should be computed in processor 0 and broadcasted to other processors.
2. You should have something in your code to control the maximum number of iterations allowed (**at most 500 iterations are allowed in all tests**), so that your code will not be running indefinitely.
3. The stopping criterion should be set so that the 2-norm residual of the approximate solution is reduced by 10^2 orders of magnitude, relative to the 2-norm residual of the initial guess. Let me know what initial guess you used.
4. For a test run, set the number of processors to be 4 and each processor has 3 grid points (what is n?). You will compare the convergence rate of these two methods by plotting the residual reduction rate as a function of the number of iterations.

Let G_J be the Gauss-Seidel iteration matrix, $\rho(G_J)$ be the spectral radii. The optimal SOR relaxation parameter ω might be computed as by

$$\omega = \frac{2}{1 + \sqrt{1 - \rho(G_J)^2}}.$$

Implement an SOR iteration step with the red-black Gauss-Seidel method. Test this and other possible relaxation parameters to see which one is the best. You should have a picture showing the relation between the number of iterations required for convergence and the value of ω.

Hand in: You need to hand in a hard copy of a comparison picture with **comments**. Your results should always be accompanied by suitable comments on the data you obtained and on how to interpret the results. You need to hand in a hard copy of your code and e-mail me your electronic code.