

# Midterm: CS685-002, Spring 2001

Time: 10:00am – 10:50am, March 6, 2001

You can use books and notes to help you answer the questions. You are not allowed to talk to others except to the instructor, once the test begins. Please answer the following questions in as much detail as possible. Always try to justify your answer.

1. (10 points) Using multigrid method to solve a two dimensional Poisson equation like  $u_{xx} + \epsilon u_{yy} = f$  discretized by the standard second order difference scheme. If  $\epsilon \ll 1$ , what will likely cause problem in standard multigrid method if the initial guess contains arbitrary error? What are possible strategies to fix this problem? If the initial error contain only smooth component in the  $y$  direction and oscillatory component in the  $x$  direction, is the standard multigrid method likely to work well? why?
2. (10 points) Please comment on the advantages of block Jacobi iteration versus point Jacobi iteration on a current generation parallel computer in terms of computational efficiency and parallelism. Justify your comments.
3. (10 points) Let  $V_m$  be the  $n \times m$  matrix with mutually orthogonal column vectors  $v_1, v_2, \dots, v_m$ , and  $H_m$  be the  $m \times m$  Hessenberg matrix whose nonzero entries are  $h_{ij}$  defined in the Arnoldi's method. We showed in class that

$$AV_m = V_m H_m + w_m e_m^T,$$

where  $w_m = h_{m+1,m} v_{m+1}$ , and  $e_m^T = [0, 0, \dots, 0, 1]^T$  is the  $m$ th unit vector. Show that

$$V_m^T AV_m = H_m.$$

4. (10 points) In Message-Passing Interface (MPI) library, what are the similarities and differences between GATHER and REDUCTION operations?
5. (10 points) In the reading paper *Semicoarsening Multigrid Method On Distributed Memory Machines*, how do the authors handle the robustness and maintain parallelism in their multigrid solver? How do they generate the coarse grids?