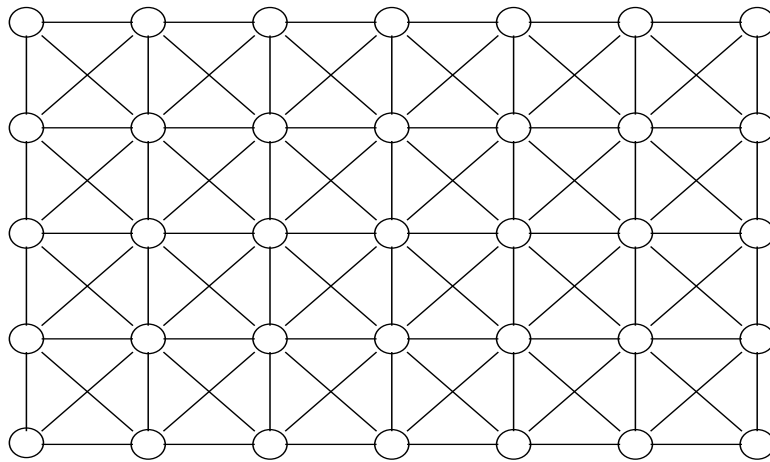


# Final Exam: CS685-002, Spring 2001

Time: 1:00pm – 3:00pm, May 4, 2001

You can use books and notes to help you answer the questions. You are not allowed to talk to others except to the instructor, once the test begins. Please answer the following questions in as much detail as possible. Always try to justify your answer.

1. (10 points) Suppose the two dimensional Poisson equation is discretized by the 9 point stencil. You can color the grid with four colors so that grid points with the same color are not connected. Please write the coefficient matrix in a block format corresponding to the four-colored ordering. (There is no need to write out the individual matrix entries, just write a block matrix with some special blocks.) Then write a Gauss-Seidel (SOR) preconditioning procedure in block format, so that each substep can be processed in parallel. Please comment on how parallelism is extracted from this multicoloring strategy.



**9-point discretization**

2. (10 points) Let a lower triangular matrix be

$$L = \begin{bmatrix} l_{11} & & & & & & \\ l_{21} & l_{22} & & & & & \\ l_{31} & 0 & l_{33} & & & & \\ 0 & l_{42} & 0 & l_{44} & & & \\ l_{51} & l_{52} & 0 & 0 & l_{55} & & \\ l_{51} & 0 & l_{33} & 0 & 0 & l_{66} & \end{bmatrix}.$$

How many parallel steps are needed to perform a forward elimination with  $Lx = b$ ? Please also indicate which variables are eliminated in which step.

3. (10 points) Let  $M$  and  $A$  be two nonsingular matrices. In order to solve the linear system

$$Ax = b$$

more efficiently, we may solve an equivalent preconditioned system

$$M^{-1}Ax = M^{-1}b,$$

where  $M$  is called a preconditioner. Please give a few convenient (at least 3) choices of  $M$  based on the matrix  $A$  and discuss the advantages and disadvantages of different choices, including memory and construction costs and inherent parallelism.

4. (10 points) Assume that  $A$  is a nonsingular matrix written in a two by two block format with

$$\begin{pmatrix} D_1 & E \\ F & D_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},$$

and  $D_1$  being nonsingular. Show how to obtain a (reduced) system which involves the variable  $x_2$  only. Please comment on how to solve the reduced system in order to solve the original system.

5. (10 points) Let  $A$  be a matrix arising from 5-point discretized Poisson equation with a mesh size  $h$  and  $B$  be a similar matrix from similar discretization with a mesh size  $2h$ . Let  $V^T$  be a given interpolation operator from the grid of  $2h$  to that of  $h$ , and  $V$  be a given restriction operator from  $h$  to  $2h$ . Design an algorithm to solve

$$Ax = b$$

using Conjugate Gradient method with the matrix  $B$  as the preconditioner. There is no need to write out a detailed CG computational steps. You just need to show how the solution is updated and how the error is corrected.