Pease show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Consider the problem \( x' = x \). If the initial condition is \( x(0) = c \), then the solution is \( x(t) = ce^t \). If a roundoff error of \( \epsilon \) occurs in reading the value of \( c \) into the computer, what effect is there on the solution at point \( t = 10 \)? At \( t = 20 \)? Do the same for \( x' = -x \).

2. (10 points) Find a polynomial \( p \) with the property \( p - p' = t^3 + t^2 - 2t \).

3. (10 points) The function \( f(x, y) = xe^y \) can be approximated by the Taylor series in two variables by \( f(x + h, y + k) \approx (Ax + B)e^y \). Determine \( A \) and \( B \) when terms through the second partial derivatives are used in the series.

4. (10 points) Solve the differential equation

\[
\begin{cases}
\frac{dx}{dt} = -tx^2 \\
x(0) = 2
\end{cases}
\]

at \( t = -0.2 \), correct to two decimal places, using one step of the Taylor series method of order 2 and one step of the Runge-Kutta method of order 2.

5. (10 points) Solve the differential equation \( x' = x \) with initial value \( x(0) = 1 \) by the Taylor series method on the interval \([0, 10]\). Compare the result with the exact solution \( x(t) = e^t \). Use derivatives up to and including the tenth. Use step size \( h = 1/100 \).

6. (10 points) Consider the third-order Runge-Kutta method:

\[
x(t + h) = x(t) + \frac{1}{9}(2K_1 + 3K_2 + 4K_3)
\]

where

\[
\begin{align*}
K_1 &= hf(t, x) \\
K_2 &= hf(t + \frac{1}{2}h, x + \frac{1}{2}K_1) \\
K_3 &= hf(t + \frac{3}{4}h, x + \frac{3}{4}K_2)
\end{align*}
\]

Show that it agrees with the Taylor series method of the same order for the differential equation \( x' = x + t \).