Homework 6: CS537, Spring 2016
Due Date: 10:50am, April 11, 2016

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Find the best function (in the least-squares sense) that fits these data points and is of the form \( f(x) = a \sin \pi x + b \cos \pi x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>-1/2</th>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

2. (10 points) An experiment involves two independent variables \( x \) and \( y \) and one dependent variable \( z \). How can a function \( z = ax + by + cy \) be fitted to the table of points \((x_k, y_k, z_k)\)? Give the normal equations.

3. (10 points) Show that the normal equations for the least-squares solution of \( Ax = b \) can be written as \((A^T A)x = A^T b\).

4. (10 points) There is a function of the form \( f(x) = \alpha x^{12} + \beta x^{13} \)

   for which \( f(0.1) = 6 \times 10^{-13} \) and \( f(0.9) = 3 \times 10^{-2} \). What is the function? Are \( \alpha \) and \( \beta \) sensitive to perturbations in the two given values of \( f(x)\)?

5. (10 points) Consider polynomials \( g_0, g_1, \ldots, g_n \) defined by \( g_0(x) = 1, g_1(x) = x - 1 \), and \( g_j(x) = 3x g_{j-1}(x) + 2g_{j-2}(x) \). Develop an efficient algorithm for computing values of the function \( f(x) = \sum_{j=0}^{n} c_j g_j(x) \).

6. (10 points) Show that the Chebyshev polynomials \( T_0, T_2, T_4, \ldots \) are even and that \( T_1, T_3, \ldots \) are odd functions. Recall that an even function satisfies the equation \( f(x) = f(-x) \); and odd function satisfies the equation \( f(x) = -f(-x) \),