

Homework 6: CS537, Fall 2008

Due Date: 3:15pm, November 18, 2008

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Define $f(x) = 0$ if $x < 0$ and $f(x) = x^2$ if $x \geq 0$. Show that f and f' are continuous. Show that any quadratic spline with knots t_0, t_1, \dots, t_n is of the form

$$ax^2 + bx + c + \sum_{i=1}^{n-1} d_i f(x - t_i)$$

2. (10 points) Let S be a cubic spline that has knots $t_0 < t_1 < \dots < t_n$. Suppose that on the two intervals $[t_0, t_1]$ and $[t_2, t_3]$, S reduces to linear polynomials. What can be said of S on $[t_1, t_2]$.
3. (10 points) Show that the indefinite integral of a first-degree spine is a second-degree spline.
4. (10 points) Determine the parameters a, b, c, d and e so that S is a natural cubic spline

$$S(x) = \begin{cases} a + b(x-1) + c(x-1)^2 + d(x-1)^3 & x \in [0, 1] \\ (x-1)^3 + ex^2 - 1 & x \in [1, 2] \end{cases}$$

5. (10 points) Determine the coefficients so that the function

$$S(x) = \begin{cases} x^2 + x^3 & 0 \leq x \leq 1 \\ a + bx + cx^2 + dx^3 & 1 \leq x \leq 2 \end{cases}$$

is a cubic spline and has the property $S_1'''(x) = 12$.