Homework 5: CS537, Spring 2016
Due Date: 10:50am, April 1, 2016

Pease show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Define \( f(x) = 0 \) if \( x < 0 \) and \( f(x) = x^2 \) if \( x \geq 0 \). Show that \( f \) and \( f' \) are continuous. Show that any quadratic spline with knots \( t_0, t_1, \ldots, t_n \) is of the form

\[
ax^2 + bx + c + \sum_{i=1}^{n-1} d_i f(x - t_i)
\]

2. (10 points) Let \( S \) be a cubic spline that has knots \( t_0 < t_1 < \cdots < t_n \). Suppose that on the two intervals \([t_0, t_1]\) and \([t_2, t_3]\), \( S \) reduces to linear polynomials. What can be said of \( S \) on \([t_1, t_2]\) (linear, quadratic, or cubic).

3. (10 points) Under what conditions that the indefinite integral of a first-degree spline is a second-degree spline.

4. (10 points) Determine the parameters \( a, b, c, d \) and \( e \) so that \( S \) is a natural cubic spline

\[
S(x) = \begin{cases} 
  a + b(x - 1) + c(x - 1)^2 + d(x - 1)^3 & x \in [0, 1] \\
  (x - 1)^3 + ex^2 - 1 & x \in [1, 2]
\end{cases}
\]

5. (10 points) Determine the coefficients so that the function

\[
S(x) = \begin{cases} 
  x^2 + x^3 & 0 \leq x \leq 1 \\
  a + bx + cx^2 + dx^3 & 1 \leq x \leq 2
\end{cases}
\]

is a cubic spline and has the property \( S''(x) = 12 \).

6. (10 points) Suppose that \( f(0) = 1, f(1) = 1.562, f'(0) = 2, \) and \( f'(1) = 2.134 \). Determine the cubic interpolating polynomial \( p_3(x) \) for these data. Is it a natural cubic spline?