

# Homework 5: CS537, Fall 2008

Due Date: 3:15pm, October 30, 2008

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Prove that the product of two unit lower triangular matrices is also unit lower triangular.
2. (10 points) Let  $A$  be a matrix of tridiagonal form such that  $a_i c_i > 0$  for  $1 \leq i \leq n-1$ . Find the general form of the diagonal matrix  $D = \text{diag}(\alpha_i)$  with  $\alpha_i \neq 0$  such that  $D^{-1}AD$  is symmetric. What is the general form of  $D^{-1}AD$ ?
3. (10 points) For what value of  $\alpha$  does the straightforward Gaussian elimination produce erroneous answers for this system?

$$\begin{aligned}x_1 + x_2 &= 2 \\ \alpha x_1 + x_2 &= 2 + \alpha\end{aligned}$$

Explain what happens in the computer.

4. (10 points) Using the Jacobi, Gauss-Seidel, and the SOR ( $\omega = 1.4$ ) iterative methods, write and run a code to solve the following linear system to four decimal places of accuracy

$$\begin{bmatrix} 7 & 3 & -1 & 2 \\ 3 & 8 & 1 & -4 \\ -1 & 1 & 4 & -1 \\ 2 & -4 & -1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -3 \\ 1 \end{bmatrix}$$

Compare the number of iterations in each case and plot a graph showing the change of the residual norms as a function of the number of iterations. The exact solution is  $x = (-1, 1, -1, 1)^T$ .

5. (10 points) Solve the following system using Gaussian elimination with scaled partial pivoting. Carry four significant digits.

$$\begin{aligned}3x_1 + 2x_2 - x_3 &= 7 \\ 5x_1 + 3x_2 + 2x_3 &= 4 \\ -x_1 + x_2 - 3x_3 &= -1\end{aligned}$$