Homework 4: CS537, Fall 2008

Due Date: 3:15pm, October 14, 2008

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Find an approximate value of $\int_1^2 x^{-1}\,dx$ using composite Simpson's rule with $h = 0.25$. Give a bound on the error. Then calculate the exact value of the integration and compute the exact error to see if the error bound is accurate.

2. (10 points) A numerical integration scheme that is not as well known is the basic Simpson's $\frac{3}{8}$ rule over three intervals

$$\int_a^{a+3h} f(x)\,dx \approx \frac{3h}{8}[f(a) + 3f(a+h) + 3f(a+2h) + f(a+3h)].$$

   Estimate the error term for this rule and explain why this rule is not as popular as the Simpson's rule.

3. (10 points) In the Romberg algorithm, $R(n, 0)$ denotes an estimate of

$$\int_a^b f(x)dx,$$

   with subintervals of size $h = (b-a)/2^n$. If it were known that

$$\int_a^b f(x)dx = R(n,0) + a_3h^3 + a_6h^6 + \cdots$$

   how would we have to modify the Romberg algorithm?

4. (10 points) What is a reasonable bound on the error when we use the composite trapezoid rule on

$$\int_0^4 \cos x^3 dx$$

   taking 201 equally spaced points (including endpoints)?

5. (10 points) Construct a rule of the form

$$\int_{-1}^1 f(x)dx \approx \alpha f(-\frac{1}{2}) + \beta f(0) + \gamma f(\frac{1}{2})$$

   that is exact for all polynomials of degree ≤ 2; that is; determine values for α, β, and γ. Hint: Make the relation exact for 1, x, and x^2 and find a solution of the resulting equations. If it is exact for these polynomials, it is exact for all polynomials of degree ≤ 2.