Homework 3: CS537, Spring 2015
Due Date: 10:00am, March 4, 2015

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Find an approximate value of \( \int_{1}^{2} \sin x \, dx \) using composite Simpson’s rule with \( h = 0.25 \). Give a bound on the error. Then calculate the exact value of the integration and compute the exact error to see if the error bound is accurate.

2. (10 points) A numerical integration scheme that is not as well known is the basic Simpson’s \( \frac{3}{8} \) rule over three intervals

\[
\int_{a}^{a+3h} f(x) \, dx \approx \frac{3h}{8} [f(a) + 3f(a + h) + 3f(a + 2h) + f(a + 3h)].
\]

Estimate the error term for this rule and explain why this rule is not as popular as the Simpson’s rule.

3. (10 points) In the Romberg algorithm, \( R(n, 0) \) denotes an estimate of \( \int_{a}^{b} f(x) \, dx \), with subintervals of size \( h = (b - a)/2^n \). If it were known that

\[
\int_{a}^{b} f(x) \, dx = R(n, 0) + a_3 h^3 + a_6 h^6 + \cdots
\]

how would we have to modify the Romberg algorithm?

4. (10 points) What is a reasonable bound on the error when we use the composite trapezoid rule on

\[
\int_{0}^{4} \cos x^3 \, dx
\]

taking 201 equally spaced points (including endpoints)?

5. (10 points) Construct a rule of the form

\[
\int_{-1}^{1} f(x) \, dx \approx \alpha f(-\frac{1}{2}) + \beta f(0) + \gamma f(\frac{1}{2})
\]

that is exact for all polynomials of degree \( \leq 2 \); that is; determine values for \( \alpha \), \( \beta \), and \( \gamma \).

6. (10 points) Determine a formula of the form

\[
\int_{0}^{h} f(x) \, dx \approx w_0 f(0) + w_1 f(h) + w_2 f''(0) + w_3 f''(h)
\]

that is exact for polynomials of as high degree as possible.