Homework 3: CS537, Fall 2008

Due Date: 3:15pm, September 30, 2008

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) There exists a unique polynomial $p(x)$ of degree 2 or less such that $p(0) = 0$, $p(1) = 1$, and $p'(\alpha) = 2$ for any value of α between 0 and 1 (inclusive), except one value of α, say α_0. Determine α_0 and give this polynomial for $\alpha \neq \alpha_0$.

2. (10 points) Can you find an approximation formula for $f'(x)$ that has error term $O(h^3)$ and involves only two evaluations of the function f? Prove and disprove.

3. (10 points) Establish the formula

$$f''(x) \approx \frac{2}{h^2}\left[\frac{f(x_0)}{(1+\alpha)} - \frac{f(x_1)}{\alpha} + \frac{f(x_2)}{\alpha(\alpha+1)}\right]$$

   in the following two ways using the unevenly spaced points $x_0 < x_1 < x_2$, where $x_1 - x_0 = h$ and $x_2 - x_1 = \alpha h$. Notice that this formula reduces to the standard central-difference formula in the notes when $\alpha = 1$.

   (a) Approximate $f(x)$ by the Newton form of the interpolating polynomial of degree 2.

   (b) Calculate the undetermined coefficient A, B, and C in the expression

$$f''(x) \approx Af(x_0) + Bf(x_1) + Cf(x_2)$$

   by making it exact for the three polynomials 1, $x - x_1$, and $(x - x_1)^2$ and thus exact for all polynomials of degree ≤ 2.

4. (10 points) Show that if a function g interpolates the function f at $x_0, x_1, \ldots, x_{n-1}$ and h interpolates f at $x_1, x_2, \ldots, x_n$, then

$$g(x) + \frac{x_0 - x}{x_n - x_0}[g(x) - h(x)]$$

   interpolates f at $x_0, x_1, \ldots, x_n$.

5. (10 points) It is suspected that the table

x	-2	-1	0	1	2	3
y	1	4	11	16	13	-4

   comes from a cubic polynomial. How can this be tested? Explain.