

## Homework 2: CS537-002, Fall 2008

Due Date: 3:15pm, September 23, 2008

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) If the bisection method is applied with starting interval  $[a, a + 1]$  and  $a \geq 2^m$ , where  $m \geq 0$ , what is the correct number of steps to compute the root with full machine precision on an IEEE single precision machine?
2. (10 points) Write a computer program to find the root of the equation

$$6(e^x - x) = 6 + 3x^2 + 2x^3$$

between  $-1$  and  $+1$  using the bisection method, and with the initial guess  $x_0 = 0.5$  using the Newton's method. Compare the rate of convergence of the two methods using a graph.

3. (10 points) Determine Newton's iteration formula for computing the cube root of  $N/M$  for nonzero integers  $N$  and  $M$ .
4. (10 points) To avoid computing the derivative at each step in Newton's method, it has been proposed to replace  $f'(x_n)$  by  $f'(x_0)$ . Derive the rate of convergence for this method.
5. (10 points) If

$$x_{n+1} = x_n + \frac{(2 - e^{x_n})(x_n - x_{n-1})}{(e^{x_n} - e^{x_{n-1}})}$$

with  $x_0 = 0$  and  $x_1 = 1$ , what is  $\lim_{n \rightarrow \infty} x_n$ ?