1. (10 points) There exists a unique polynomial \( p(x) \) of degree 2 or less such that 
\[ p(0) = 0, p(1) = 1, \] 
and \( p'(\alpha) = 2 \) for any value of \( \alpha \) between 0 and 1 (inclusive), 
except one value of \( \alpha \), say \( \alpha_0 \). Determine \( \alpha_0 \) and give this polynomial for 
\( \alpha \neq \alpha_0 \).

2. (10 points) Suppose \( \cos x \) is to be approximated by an interpolating polynomial 
of degree \( n \), using \( n + 1 \) equally spaced nodes in the interval \([0, 1]\). How accurate 
is the approximation? (Express your answer in terms of \( n \).) How accurate is the 
approximation when \( n = 9 \)? For what values of \( n \) is the error less than \( 10^{-7} \)?

3. (10 points) Establish the formula 
\[ f''(x) \approx \frac{2}{h^2} \left[ \frac{f(x_0)}{1 + \alpha} - \frac{f(x_1)}{\alpha} + \frac{f(x_2)}{\alpha(\alpha + 1)} \right] \] 
in the following two ways using the unevenly spaced points \( x_0 < x_1 < x_2 \), where 
\( x_1 - x_0 = h \) and \( x_2 - x_1 = \alpha h \). Notice that this formula reduces to the standard 
central-difference formula in the notes when \( \alpha = 1 \).

(a) Approximate \( f(x) \) by the Newton form of the interpolating polynomial of degree 
2.

(b) Calculate the undetermined coefficient \( A \), \( B \), and \( C \) in the expression 
\[ f''(x) \approx Af(x_0) + Bf(x_1) + Cf(x_2) \] 
by making it exact for the three polynomials \( 1 \), \( x - x_1 \), and \( (x - x_1)^2 \) and thus 
exact for all polynomials of degree \( \leq 2 \).

4. (10 points) Show how Richardson extrapolation would work on formula 
\[ f''(x) \approx \frac{1}{h^2} \left[ f(x + h) - 2f(x) + f(x - h) \right] \] 
to develop an expression that has a truncation error \( O(h^4) \).

5. (10 points) Find a polynomial \( p(x) \) of degree at most 3 such that 
\( p(0) = 1, p(1) = 0, \) \( p'(0) = 0, \) and \( p'(-1) = -1 \).

6. (10 points) Use a divided-difference table to show that the following data can be 
represented by a polynomial of degree 3:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>4</td>
<td>11</td>
<td>16</td>
</tr>
</tbody>
</table>