Homework 1: CS537, Spring 2015
Due Date: 11:00am, January 30, 2015

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) If the bisection method is applied with starting interval \([a, a + 1]\) and \(a = 0\), how many steps of the bisection method are needed to determine the root with an error of at most \(0.5 \times 10^{-8}\)?

2. (10 points) Write a computer program to find the root of the equation
\[
6(e^x - x) = 6 + 3x^2 + 2x^3
\]
between \(-1\) and \(+0.8\) using the bisection method, and with the initial guess \(x_0 = 0.4\) using the Newton’s method. Compare the rate of convergence of the two methods using a graph, and explain your results. (Graph the absolute function value with respect to the number of iterations and answer which one converges faster?)

3. (10 points) Determine Newton’s iteration formula for computing the cube root of \(\frac{N}{M}\) for nonzero integers \(N\) and \(M\). Do not carry out the actual computation.

4. (10 points) Find the two positive zeros of \(x^4 + 2x^3 - 7x^2 + 3\) using Newton’s method, correct to two significant figures. You need to show detailed computational steps (what is the initial guesses, the iterates, and the functional and derivative values at each step.). Just giving solutions without any details will receive zero point for this problem.

5. (10 points) Consider the Newton’s method
\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]
If the generated sequence \(\{x_n\}\) converges, then the limit point is a solution. Explain why or why no.

6. (10 points) A method of finding a zero of a given function \(f\) proceeds as follows. Two initial approximations \(x_0\) and \(x_1\) to zero are chosen, the value of \(x_0\) is fixed, and successive iterations are given by
\[
x_{n+1} = x_n - \left(\frac{x_n - x_0}{f(x_n) - f(x_0)}\right) f(x_n).
\]
This process will converge to a zero of \(f\) under certain conditions. Show that the rate of convergence to a simple zero is linear under some conditions.