

# MIDTERM EXAM II: CS321-003, Spring 2004

2:00PM-2:50PM, March 31, 2004

**This is a closed book test. Please show all steps in your work. Please finish your work independently.** (No calculator is needed for any problem).

1. (12 points) If the composite trapezoid rule is used to compute

$$\int_0^{\pi/2} \sin x \, dx$$

with an error of at most  $\pi/6 \times 10^{-4}$ , what is the uniform grid spacing  $h$ ?

2. (12 points) Suppose  $\epsilon$  is a small number close to zero,  $0 < \epsilon \ll 1$ , solve the system

$$\begin{aligned}\epsilon x_1 + x_2 &= 1 + \epsilon \\ x_1 + \epsilon x_2 &= 2\end{aligned}$$

using Gaussian elimination with scaled partial pivoting. (**Show all steps**).

3. (14 points) Let the lower sum and upper sum of

$$\int_1^2 f(x) \, dx = \int_1^2 x^{-2} \, dx$$

be  $L(f, h)$  and  $U(f, h)$  with uniform grid spacing  $h = 10^{-2}$ . If we compute

$$\int_1^2 x^{-2} \, dx \approx \frac{U(f, h) + L(f, h)}{2},$$

what is the error bound of this approximation. Hint: using a smart strategy (from one of the previous homework problems) to solve this problem.

4. (12 points) If  $\phi(h) = L - c_1 h^{1/2} - c_2 h^{2/2} - c_3 h^{3/2} - \dots$ , then what combination of  $\phi(h)$  and  $\phi(h/2)$  should give an accurate estimate of  $L$ ?

**Please remember to write down your name on the answer sheets.**