Regular problems

1. (4 points) Convert the decimal number \( (47.15)_{10} \) into an octal number.
   A. \((45.143...)_8\).
   B. \((92.176...)_8\).
   C. \((57.114...)_8\).
   D. \((61.312...)_8\).

2. (4 points) If we use bisection method to find a root of the function \( f(x) = \exp x - x^2 - 1.5 \) on interval \([-1, 3]\), at least how many steps are needed if we want the difference between the approximate root and the true root to be smaller than \( 2^{-15} \)? (You do not need to compute the root.)
   A. 9 steps.
   B. 16 steps.
   C. 21 steps.
   D. 76 steps.

3. (4 points) Construct a Newton's interpolation polynomial for the data shown
   
   \[
   \begin{array}{c|cccc}
   x & 0 & 1 & 2 & 3 \\
   \hline
   y & 7 & 11 & 13 & 15 \\
   \end{array}
   \]
   using divided-difference table.
   A. \( p_3(x) = 7 + 4x - x(x - 1) + \frac{1}{3}x(x - 1)(x - 2) \).
   B. \( p_3(x) = 0 + 7x + 4x(x - 1) - x(x - 1)(x - 2) \).
   C. \( p_3(x) = 11 + 4(x - 1) - (x - 1)(x - 2) + \frac{1}{3}(x - 1)(x - 2)(x - 3) \).
   D. \( p_3(x) = x + 11x + 13x(x - 1) + 15x(x - 1)(x - 2) \).

4. (4 points) Using Taylor's formula, we can derive an approximation formula in the form of
   \[ f'(x) \approx \frac{f(x + 2h) - f(x - 2h)}{4h} \]
   The leading truncation error of this approximation formula is
   A. \( \frac{1}{3}h^2f''(x) \).
5. (4 points) Consider a computer that uses four-decimal-digit numbers and correct rounding. Let $\text{fl}(x)$ denote the floating-point machine number closest to $x$. If $x = 0.3271653$ and $y = 0.3269104$, compute the result of the operation $\text{fl}(x) - \text{fl}(y)$ (Use correct rounding).

A. The result is $0.51 \times 10^{-3}$.
B. The result is $0.2549 \times 10^{-3}$.
C. The result is $0.2 \times 10^{-3}$.
D. The result is $0.3 \times 10^{-3}$.

**Bonus Problems:** These problems are 2 points each, and can be answered to get additional points for this exam, only if you do not get full scores for the previous problems. The total scores that you can get from this final exam is 20.

**Bonus1 (2 points)** When you choose a numerical method to find the root of a continuous function $f(x)$, which of the following is most appropriate in terms of robustness and convergence rate:

A. Bisection method, because it always converges in such a case.
B. Newton’s method, because it has a quadratic convergence and only needs one function evaluation.
C. Secant method, because it does not need the derivative and converges super-linearly.
D. Bisection method, because it does not need the derivative and converges linearly.

**Bonus2 (2 points)** For some values of $x$, the function $f(x) = e^x - \cos x - \sin x$ cannot be accurately computed by using this formula. Explain the reason and find a way around the difficulty.

A. $x \approx \frac{\pi}{2}$, the original formula will be inaccurate because of loss of significant digits. we can use Taylor series and get an approximate formula as $f(x) \approx \frac{x^2}{2!} + \frac{x^3}{3!}$.
B. $x \approx 0$, the original formula will be inaccurate because of loss of significant digits. we can use Taylor series and get an approximate formula as $f(x) \approx x^2 + \frac{x^3}{3}$.
C. $x \approx 0$, the original formula will be inaccurate because of loss of significant digits. we can use rationalization and get a formula as $f(x) = \frac{1}{e^x + \sin x + \cos x}$.
D. $x \approx \pi$, the original formula will be inaccurate because of loss of significant digits. we can use Taylor series and get an approximate formula as $f(x) \approx \frac{x^2}{2} + \frac{x^3}{3}$.
Your Name:

Please give only one letter to represent your answer to each problem.

Regular problems

1. The answer is: ( ).
2. The answer is: ( ).
3. The answer is: ( ).
4. The answer is: ( ).
5. The answer is: ( ).

Bonus problems

Bonus1 The answer is ( ).
Bonus2 The answer is ( ).