

# Homework 5: CS321, Spring 2007

Due Date: 2:50pm, April 11, 2007

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Find an approximate value of  $\int_1^2 x^{-1} dx$  using composite Simpson's rule with  $h = 0.25$ . Give a bound on the error. Then calculate the exact value of the integration and compute the exact error to see if the error bound is accurate.
2. (10 points) A numerical integration scheme that is not as well known is the basic Simpson's  $\frac{3}{8}$  rule over three intervals

$$\int_a^{a+3h} f(x) dx \approx \frac{3h}{8}[f(a) + 3f(a+h) + 3f(a+2h) + f(a+3h)].$$

Estimate the error term for this rule and explain why this rule is not as popular as the Simpson's rule.

3. (10 points) For what value of  $\alpha$  does the straightforward Gaussian elimination produce erroneous answers for this system?

$$\begin{aligned}x_1 + x_2 &= 2 \\ \alpha x_1 + x_2 &= 2 + \alpha\end{aligned}$$

Explain what happens in the computer.

4. (10 points) Solve the following system using straightforward Gaussian elimination with forward elimination and back substitution. Carry four significant digits.

$$\begin{aligned}3x_1 + 2x_2 - 5x_3 &= 0 \\ 2x_1 - 3x_2 + x_3 &= 0 \\ x_1 + 4x_2 - x_3 &= 4\end{aligned}$$

5. (10 points) Solve the following system using Gaussian elimination with scaled partial pivoting. Carry four significant digits.

$$\begin{aligned}3x_1 + 2x_2 - x_3 &= 7 \\ 5x_1 + 3x_2 + 2x_3 &= 4 \\ -x_1 + x_2 - 3x_3 &= -1\end{aligned}$$