

Homework 4: CS321, Spring 2007

Due Date: 1:50pm, March 19, 2007

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Derive the approximation formula for the first derivative

$$f'(x) \approx \frac{1}{2h}[4f(x+h) - 3f(x) - f(x+2h)]$$

and show that its error term is of the form $\frac{1}{3}h^2 f'''(\xi)$.

2. (10 points) Establish the formula for the second derivatives

$$f''(x) \approx \frac{1}{4h^2}[f(x+2h) - 2f(x) + f(x-2h)]$$

and establish the term for the error in using this approximation formula.

3. (10 points) If

$$\phi(h) = L - c_1 h - c_2 h^2 - c_3 h^3 - \dots,$$

then what combination of $\phi(h)$ and $\phi(h/2)$ should give an accurate estimate of L ?

4. (10 points) For a decreasing function $f(x)$ over an interval $[a, b]$ with n uniform subintervals, show that the difference between the upper sum and the lower sum is given by $[(b-a)/n][f(a) - f(b)]$.
5. (10 points) Compute two approximate values for $\int_1^2 dx/x^2$ using $h = 1/2$ with lower sums and the composite trapezoid rule.