

## Homework 3: CS321, Spring 2007

Due Date: 1:50pm, March 2, 2007

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Verify that the polynomials

$$\begin{aligned}p(x) &= 5x^3 - 27x^2 + 45x - 21 \\q(x) &= x^4 - 5x^3 + 8x^2 - 5x + 3\end{aligned}$$

interpolate the data

$x$	1	2	3	4
$y$	2	1	6	47

and explain why this does not violate the uniqueness part of the theorem on existence of polynomial interpolation.

2. (10 points) Given the data

$x$	0	1	2	4	6
$f(x)$	1	9	23	93	259

do the following

- Construct the divided-difference table.
  - Using Newton's interpolation polynomial, find an approximation to  $f(4.2)$ .
3. (10 points) Show that if  $f$  is continuously differentiable on the interval  $[x_0, x_1]$ , then  $f[x_0, x_1] = f'(c)$  for some  $c$  in  $(x_0, x_1)$ .
4. (10 points) Show that if a function  $g$  interpolates the function  $f$  at  $x_0, x_1, \dots, x_{n-1}$  and  $h$  interpolates  $f$  at  $x_1, x_2, \dots, x_n$ , then

$$g(x) + \frac{x_0 - x}{x_n - x_0}[g(x) - h(x)]$$

interpolates  $f$  at  $x_0, x_1, \dots, x_n$ .

5. (10 points) Construct an inverse interpolation to solve equations of the form  $f(x) = 0$ . Test it on the data to find an approximate root

$y$	1	2	3	-4	5
$x$	2	48	272	1182	2262