Homework 2: CS321, Fall 2015
Due Date: 3:15PM, September 22, 2015

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) If \( a = 0.1 \) and \( b = 1.0 \), how many steps of the bisection method are needed to determine the root with an error of at most \( \frac{1}{2} \times 10^{-8} \)?

2. (10 points) The equation \( x - Rx^{-1} = 0 \) has \( x = \pm R^{1/2} \) for its solution. Establish the Newton’s iterative scheme, in simplified form, for this situation (to compute the root). There is no need to carry out the actual computation.

3. (10 points) Find the root of the equation
\[
x^3 = x^2 + x + 1
\]
in the interval \([1, 3]\) by Newton’s method using double precision. Use \( x_0 = 1.5 \) and iterate 5 steps. Make a table that shows the number of correct digits in each step. You need also submit your code with your homework.

4. (10 points) If the secant method is used on \( f(x) = x^5 + x^3 + 3 \) and if \( x_{n-2} = 0 \) and \( x_{n-1} = 1 \), what is \( x_n \)?

5. (10 points) Consider the following iteration procedure
\[
x_{n+1} = \frac{1}{2} x_n + \frac{1}{x_n}.
\]
Does it converge for any nonzero initial point? If not, why. If so, to what value?

6. (10 points) For the bisection method starting with an interval \([a_0, b_0]\), and generating a sequence of bisecting middle points \( \{c_0, c_1, \ldots, c_n, \ldots\} \), we have
\[
b_n - a_n = \frac{b_0 - a_0}{2^n}, \ldots \text{ and } c_n = \frac{b_n + a_n}{2}.
\]
Show that the sequence \( \{c_n\} \) satisfies
\[
|c_n - c_{n+1}| \leq \frac{b_0 - a_0}{2^{n+2}}.
\]