

FINAL EXAM: CS321-002, Fall 2000

(Please show all steps)

- (5 points) Convert $(0.372)_{10}$ into an octal base number with the first 3 nonzero digits.
- (5 points) If the Gaussian elimination algorithm with scaled partial pivoting is used on the matrix below, what is the scale vector? What is the first pivot row? (Do not solve the system.)

$$\begin{bmatrix} 4 & 7 & 3 \\ 1 & 3 & 2 \\ 2 & -4 & -1 \end{bmatrix}$$

- (5 points) What condition is sufficient to guarantee that there is no pivoting is needed to solve a tridiagonal linear system? Is the following tridiagonal matrix satisfying the condition?

$$\begin{bmatrix} 2 & 1 & & & \\ 2 & 5 & 2 & & \\ & 1 & 3 & 1 & \\ & & 3 & 1 & 1 \\ & & & 1 & 1 \end{bmatrix}$$

- (5 points) Use the composite trapezoid rule with 0, 1, 2 as partition points to compute

$$\int_0^2 (x^2 + 1) dx$$

- (10 points) Find the linear least squares approximation to the data

$$\begin{array}{c|ccc} x & 1 & 2 & 3 \\ \hline y & 1 & 4 & 8 \end{array}$$

Note that you need to find the linear function and compute the total error.

- (10 points) Determine the coefficients a, b, c and d in the function

$$S(x) = \begin{cases} x^3 - 1 & -9 \leq x \leq 0 \\ ax^3 + bx^2 + cx + d & 0 \leq x \leq 5 \end{cases}$$

so that it is a cubic spline that takes the value 2 when $x = 1$.

- (10 points) Verify that when Newton's method is used to compute \sqrt{R} for some real number R (by solving the equation $x^2 = R$), the sequence of iterates is defined by

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{R}{x_n} \right).$$

Show that the sequence defined above satisfies

$$x_{n+1}^2 - R = \left[\frac{x_n^2 - R}{2x_n} \right]^2.$$