Homework 3: CS321-002, Fall 2000

Answer Sheet

1. (10 points) For this problem, we need to verify the definition of polynomial interpolation, i.e., we need to verify that

$$p(x_i) = y_i, \qquad q(x_i) = y_i, \qquad i = 0, 1, 2, 3.$$

It turns that both $p(x)$ and $q(x)$ interpolate the table.

The Theorem says that there is a unique polynomial of degree less or equal to n interpolates a table with $n+1$ data sets. For this problem, $n = 3$. But $q(x)$ is of order 4. So the Theorem does not apply. In other words, the Theorem is not violated, but its condition is violated.

2. (10 points)
The divided difference table is

x	$f[\,]$	$f[\,,\,]$	$f[\,,\,,\,]$	$f[\,,\,,\,,\,]$	$f[\,,\,,\,,\,,\,]$
0	1				
		8			
1	9		3		
		14		1	
2	23		7		0
		35		1	
4	93		12		
		83			
6	259				

The interpolation polynomial is

$$p_4(x) = 1 + 8x + 3x(x-1) + x(x-1)(x-2)$$

So we have

$$f(4.2) \approx p_4(4.2) = 104.488.$$

3. (10 points) For the closed interval $[x_0, x_1]$, the Mean Value Theorem states that there exists a point $\xi \in (x_0, x_1)$ such that

$$f'(\xi) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

By the definition of divided difference, the right-hand side of the above equation is nothing but $f[x_0, x_1]$. So we have $f'(\xi) = f[x_0, x_1]$ for some $\xi \in (x_0, x_1)$.

4. (10 points) You need to make sure you understand the definition of polynomial interpolation.

Since g interpolates f at $x_0, x_1, \ldots, x_{n-1}$, we have $g(x_i) = f(x_i)$ for $0 \le i \le (n-1)$. Similarly, h interpolating f at $x_1, x_2, \ldots, x_n$ implies that $h(x_i) = f(x_i)$ for $1 \le i \le n$. Hence, $g(x_1) = h(x_i)$ for $1 \le i \le (n-1)$.

Now let

$$k(x) = g(x) + \frac{x_0 - x}{x_n - x_0}\,[g(x) - h(x)]$$

We want to show that $k(x_i) = f(x_i)$ for $0 \le i \le n$.

First verify at x_0

$$k(x_0) = g(x_0) + \frac{x_0 - x_0}{x_n - x_0}\,[g(x_0) - h(x_0)] = g(x_0) = f(x_0).$$

For $1 \le i \le (n-1)$, we have

$$k(x_i) = g(x_i) + \frac{x_0 - x_i}{x_n - x_0}\,[g(x_i) - h(x_i)] = f(x_i) + \frac{x_0 - x_i}{x_n - x_0}\,[f(x_i) - f(x_i)] = f(x_i)$$

For point x_n, we have

$$k(x_n) = g(x_n) + \frac{x_0 - x_n}{x_n - x_0}\,[g(x_n) - h(x_n)] = h(x_n) = f(x_n).$$

Since $k(x_i) = f(x_i)$ for $i = 0, 1, 2, \ldots, x_n$, we conclude that $k(x)$ interpolates $f(x)$ at $x_0, x_1, x_2, \ldots, x_n$.

5. (10 points) A computer code is preferred for this problem. However, we can construct a divided difference table as

y	$g[\,]$	$g[\,,\,]$	$g[\,,\,,\,]$	$g[\,,\,,\,,\,]$	$g[\,,\,,\,,\,,\,]$
1	2				
		46			
2	48		89		
		224		6	
3	272		59		4
		−130		22	
−4	1182		125		
		120			
5	2262				

The interpolation polynomial is

$$p_4(y) = 2 + 46(y-1) + 89(y-1)(y-2) + 6(y-1)(y-2)(y-3) + 4(y-1)(y-2)(y-3)(y+4).$$

Hence we have an approximate root at $x = p_4(0) = 2$.