Tally NP Sets and Easy Census Functions

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Abstract. We study the question of whether every P set has an easy (i.e., polynomial-time computable) census function. We characterize this question in terms of unlikely collapses of language and function classes such as $\#\mathrm{P}_1 \subseteq \mathrm{FP}$, where $\#\mathrm{P}_1$ is the class of functions that count the witnesses for tally NP sets. We prove that every $\#\mathrm{P}_1^{\mathrm{PH}}$ function can be computed in $\mathrm{FP}^{\#\mathrm{P}_1^{\#\mathrm{P}_1}}$. Consequently, every P set has an easy census function if and only if every set in the polynomial hierarchy does. We show that the assumption $\#\mathrm{P}_1 \subseteq \mathrm{FP}$ implies $\mathrm{P} = \mathrm{BPP}$ and $\mathrm{PH} \subseteq \mathrm{MOD}_k\mathrm{P}$ for each $k \geq 2$, which provides further evidence that not all sets in P have an easy census function. We also relate a set's property of having an easy census function to other well-studied properties of sets, such as rankability and scalability (the closure of the rankable sets under P-isomorphisms). Finally, we prove that it is no more likely that the census function of any set in P can be approximated (more precisely, can be n^{α}-enumerated in time n^{β} for fixed α and β) than that it can be precisely computed in polynomial time.

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