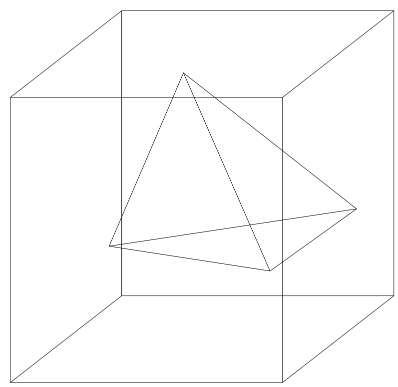
CS 633 3D Computer Animation

Solution Set - HW 6 (40 points)

Due: 3/29/07

1. Given two *star-shaped* polyhedra, how would you modify their vertex-edge-face structures (boundary representations) so that they would have the same topology (so that we can perform interpolation on corresponding vertices to generate intermediate shapes in the process of changing one object into the other)? Use the following cube and the enclosed pyramid to explain your concept. Remember, at the end, the modified boundary representations should have the same vertex-edge-face structure. (10 points)



Sol.

1. First, find a point that is inside both objects (such as the mid-point of the centroids of the objects), call that point *O*.
2. For each vertex of the pyramid, generate a ray from *O* to that vertex and find its intersection point with the cube. Do the same thing for vertices of the cube.
3. If a face of the pyramid does not contain any of the new intersection points, keep that face. Otherwise, use the intersection points to break the face into subfaces. For instance, if the face contains one intersection point, then use that intersection point and vertices of the face to form three new subfaces. If the face contains two intersection points, first use one of the intersection points to break the face into three subfaces. One of the subfaces would contain the second intersection point as an interior point. Then use the second intersection point to break that subface into three sub-sub-faces.
4. At this point, the new vertex-edge-face structures of the cube and the pyramid are the same.

2. Question: can any genus-0 polyhedron be mapped to a sphere? If your answer is YES, prove it (must be precise). Otherwise, use an example to explain why this is not possible. (20 points)

Sol.

The answer is YES. The following expansion algorithm can be used to construct such a mapping.

1. First construct a skelton of the given genus-0 polyhedron. (What is a skelton of a polyhedron?)
2. Set r to be one half of the largest radius in constructing the skelton
2. Repeat the the following process until the polyhedron is a good approximation of a sphere:
   (1) Traverse the the skelton of the polyhedron with a sphere of reaius r. For each point of the skelton, if r is bigger than the radius of the sphere for that point, expand the polyhedron (how?) of the polyhedron around that point until radius of the sphere for that point is $\geq r$.
   (2) Increment r by a pre-set step size, then repeat the above step.

3. Use a drawing to show the 'reachable workspace' of the following robot arm. Here we assume $|L_2| = |L_3| = |L_1|/4$, θ_2 and θ_3 can be any value, and $-\pi/2 \leq \theta_1 \leq \pi/2$. (10 points)

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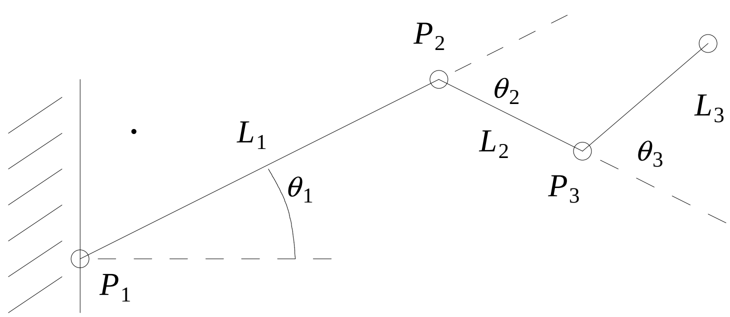
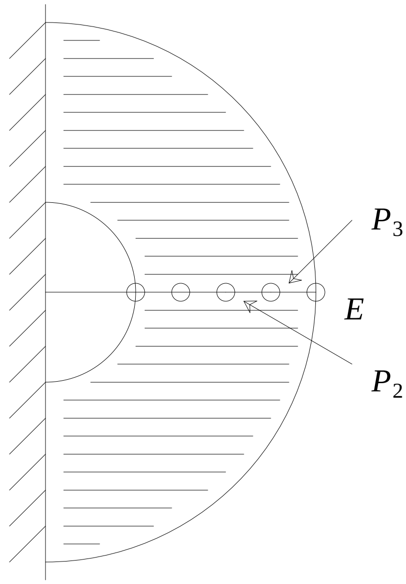


Sol.



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P_2

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