CS 633 3D Computer Animation

Solution Set - HW 2 (40 points)

1. I have shown you in class how to find the control points of a uniform cubic B-spline curve that interpolates six given data points $\mathbf{D}_0$, $\mathbf{D}_1$, $\mathbf{D}_2$, $\mathbf{D}_3$, $\mathbf{D}_4$ and $\mathbf{D}_5$ (page 24 of the Notes). This curve has five segments. So, it needs eight control points $\mathbf{P}_0$, $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$, $\mathbf{P}_5$, $\mathbf{P}_6$ and $\mathbf{P}_7$. To find these control points, you need to solve a system of four equations to find $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$ and $\mathbf{P}_5$ first, and then compute $\mathbf{P}_0$ and , $\mathbf{P}_7$. You don't have to compute $\mathbf{P}_1$ and $\mathbf{P}_6$ because they are equal to $\mathbf{D}_0$ and $\mathbf{D}_5$, respectively. Now, here is the question: to build a uniform cubic B-sline curve to interpolate nine data points: $\mathbf{D}_0$, $\mathbf{D}_1$, ..., $\mathbf{D}_9$ and $\mathbf{D}_{10}$, how many segments should it have? and how many control points should this curve have?

Sol.

10 segments, 13 control points.

2. To build a uniform cubic B-sline curve to interpolate 11 data points: $\mathbf{D}_0$, $\mathbf{D}_1$, ..., $\mathbf{D}_9$ and $\mathbf{D}_{10}$, we also need to solve a system of equations for some of the control points. How big is this system (i.e., how many equations are there in this system)? Which control points (using the notations of Question 1) will be computed from this system?

Sol.

A system of 9 equations. The control points to be solved from these equations are $\mathbf{P}_2$, $\mathbf{P}_3$, ..., $\mathbf{P}_9$ and $\mathbf{P}_{10}$.

The remaining control points are computed as follows:

$$\mathbf{P}_1 = \mathbf{D}_0, \quad \mathbf{P}_{11} = \mathbf{D}_{10}$$

$$\mathbf{P}_0 = 2\mathbf{P}_1 - \mathbf{P}_2, \quad \mathbf{P}_{12} = 2\mathbf{P}_{11} - \mathbf{P}_{10}$$

3. Here is a more specific question. What is the exact form of the system of equations to be solved in Question 2? Give your answer in matrix form.

Sol.

The matrix form is as follows:

$$
\begin{bmatrix}
2/3 & 1/6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1/6 & 2/3 & 1/6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1/6 & 2/3 & 1/6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/6 & 2/3 & 1/6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/6 & 2/3 & 1/6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1/6 & 2/3 & 1/6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1/6 & 2/3 & 1/6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 2/3 & 1/6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 2/3
\end{bmatrix}
\begin{bmatrix}
\mathbf{P}_2 \\ \mathbf{P}_3 \\ \mathbf{P}_4 \\ \mathbf{P}_5 \\ \mathbf{P}_6 \\ \mathbf{P}_7 \\ \mathbf{P}_8 \\ \mathbf{P}_9 \\ \mathbf{P}_{10}
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{D}_1 - 1/6\mathbf{D}_0 \\ \mathbf{D}_2 \\ \mathbf{D}_3 \\ \mathbf{D}_4 \\ \mathbf{D}_5 \\ \mathbf{D}_6 \\ \mathbf{D}_7 \\ \mathbf{D}_8 \\ \mathbf{D}_9 - 1/6\mathbf{D}_{10}
\end{bmatrix}
$$

Once we have $\mathbf{P}_2$, $\mathbf{P}_3$, ... $\mathbf{P}_9$, and $\mathbf{P}_{10}$, we then use the following equations to compute the remaining control points.

$$\mathbf{P}_1 = \mathbf{D}_0, \quad \mathbf{P}_{11} = \mathbf{D}_{10}, \quad \mathbf{P}_0 = 2\mathbf{P}_1 - \mathbf{P}_{2,} \quad \mathbf{P}_{12} = 2\mathbf{P}_{11} - \mathbf{P}_{10}.$$

4. If we use the approach covered on pages 116-124, then what system of equations should we solve to get the control points, in matrix form?

Sol.

The following system:

$$
\begin{bmatrix}
3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1/4 & 7/12 & 1/6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1/6 & 2/3 & 1/6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/6 & 2/3 & 1/6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/6 & 2/3 & 1/6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1/6 & 2/3 & 1/6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1/6 & 2/3 & 1/6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 2/3 & 1/6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 2/3 & 1/6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/6 & 7/12 & 1/4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 3
\end{bmatrix}
\begin{bmatrix}
\mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \\ \mathbf{P}_4 \\ \mathbf{P}_5 \\ \mathbf{P}_6 \\ \mathbf{P}_7 \\ \mathbf{P}_8 \\ \mathbf{P}_9 \\ \mathbf{P}_{10} \\ \mathbf{P}_{11}
\end{bmatrix}
=
\begin{bmatrix}
2\mathbf{D}_0 \\ \mathbf{D}_1 \\ \mathbf{D}_2 \\ \mathbf{D}_3 \\ \mathbf{D}_4 \\ \mathbf{D}_5 \\ \mathbf{D}_6 \\ \mathbf{D}_7 \\ \mathbf{D}_8 \\ \mathbf{D}_9 \\ 2\mathbf{D}_{10}
\end{bmatrix}
$$

In this case, we have $\mathbf{P}_0 = \mathbf{D}_0$ and $\mathbf{P}_{12} = \mathbf{D}_{10}$.