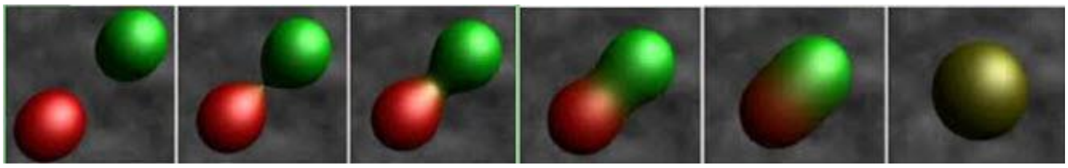
CS633 Computer Animation
Programming Assignment 3 (40 points)
Due: 5/1/2018

This programming assignment is optional. You can either do this programming assignment or do homework assignments 6 and 7.

Your task here is to use implicit surfaces to simulate the merging of two water drops (called metaballs by Jim Blinn, the developer of this technique). See the following figure for the merging process of two water drops.



The command line for the execution of your program should be as follows:

prog3   P1   P2   k

"prog3" is the name of the executible, "$P_1 = (x_1, y_1, z_1)$" is the initial location of the center of the first water drop, "$P_2 = (x_2, y_2, z_2)$" is the initial location of the center of the second water drop, and "k" is the number of intermediate images to be generated during the merging process ("k" should be at least 30).

Once executed, your program should display two separate water drops on screen, one with center P_1 and one with center P_2 (see the left most case in the above figure). These water drops are then moved toward each other to carry out a merging process. The merging process stops when the centers of the water drops coincide (see the right most case in the above figure).

The implicit representation you should use for the water drops is as follows:

$$F(x,y,z) = \sum_{i=1}^{2} \frac{1}{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2} - 1 \qquad (1)$$

where $P_i = (x_i, y_i, z_i)$ is the center of drop i.

First consider the special case that $P_1 = (0, \alpha, 0)$ and $P_2 = (0, -\alpha, 0)$ with α being a positive number. In this case equation (1) is of the following form:

$$F(x,y,z) = \frac{1}{x^2+(y-\alpha)^2+z^2} + \frac{1}{x^2+(y+\alpha)^2+z^2} + 1 \qquad (2)$$

To find intersection points of this implicit surface with the y-axis, we solve the following equation

$$F(0,y,0) = \frac{1}{(y-\alpha)^2} + \frac{1}{(y+\alpha)^2} - 1 = 0$$

This equation leads to the following equation

$$y^4 - 2(\alpha^2+1)y^2 + \alpha^2(\alpha^2-2) = 0 \quad (3)$$

If we set $x = y^2$, we have

$$x^2 - 2(\alpha^2+1)x + \alpha^2(\alpha^2-2) = 0 \quad (4)$$

Solving (4) we get two solutions for x

$$x = (\alpha^2+1) \pm \sqrt{4\alpha^2+1} \quad (5)$$

Consequently we have

$$y^2 = (\alpha^2+1) + \sqrt{4\alpha^2+1} \quad (6)$$

or

$$y^2 = (\alpha^2+1) - \sqrt{4\alpha^2+1} \quad (7)$$

From (6) and (7) we can see that (3) has four solutions when $\alpha > \sqrt{2}$

$$y = \pm\sqrt{(\alpha^2+1) + \sqrt{4\alpha^2+1}}$$
$$y = \pm\sqrt{(\alpha^2+1) - \sqrt{4\alpha^2+1}} \quad (8)$$

That is why we know equation (2) represents two disjoint components when α is large enough. This is the left most case in the above figure.

Equation (3) has three solutions when $\alpha = \sqrt{2}$

$$y = \pm\sqrt{(\alpha^2+1) + \sqrt{4\alpha^2+1}}$$
$$y = 0 \quad (9)$$

That is when the two water drops touch each other at one point, the second case from left in the above figure.

Equation (3) has two solutions when $0 \leq \alpha < \sqrt{2}$

$$y = \pm\sqrt{(\alpha^2 + 1) + \sqrt{4\alpha^2 + 1}} \tag{10}$$

This is when the two water drops merge into one drop, the four right most cases in the above figure.

For other points of the implicit surface, we can consider intersection points of (2) with vertical lines perpendicular to the xz-plane $\{(x_0, y, z_0) | \, y \in R\}$. However, for rendering purpose, we should construct a linear polygonal approximation of the implicit surface using the technique introduced in slide 8 of the notes "Special Models for Animation I" first, and then render the linear polygonal approximation.

In the general case $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$, first move the mid-point of P_1 and P_2 to the origin of the coordinate system, then perform a rotation so that the line P_1P_2 would coincide with the y-axis. After this, we can use the above approach to get a linear polygonal approximation of the transformed implicit surface and then transform the linear polygonal approximation back to the original location (and orientation) of the implicit surface to do rendering. This is done for each of the k intermediate images.