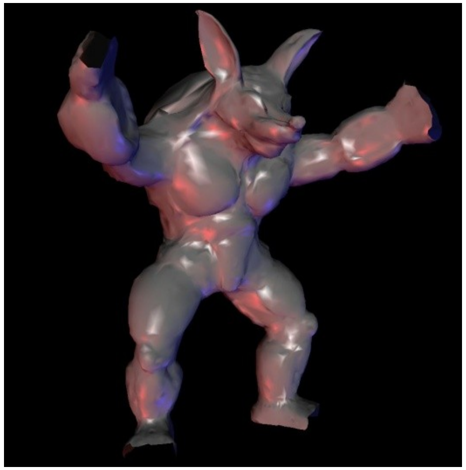
12.3 Subdivision Surfaces

What is subdivision based representation?

Subdivision Surfaces



What is so special?

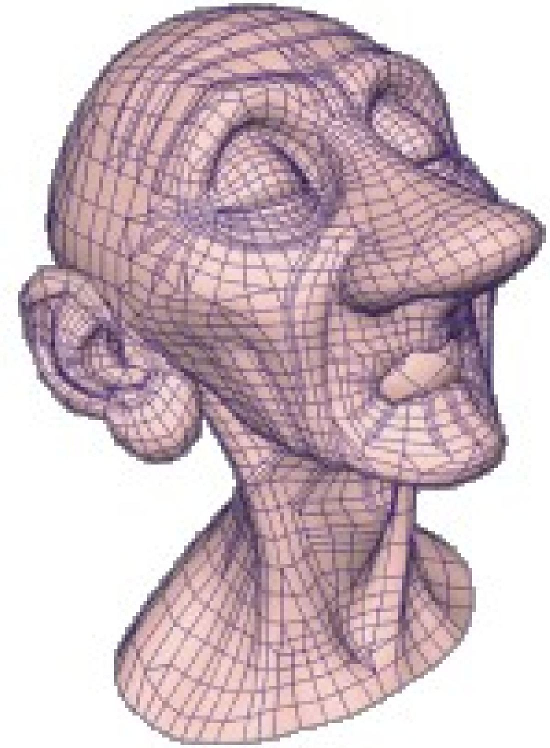
Multi-resolution
(Scalability)

One piece representation™
(arbitrary topology)

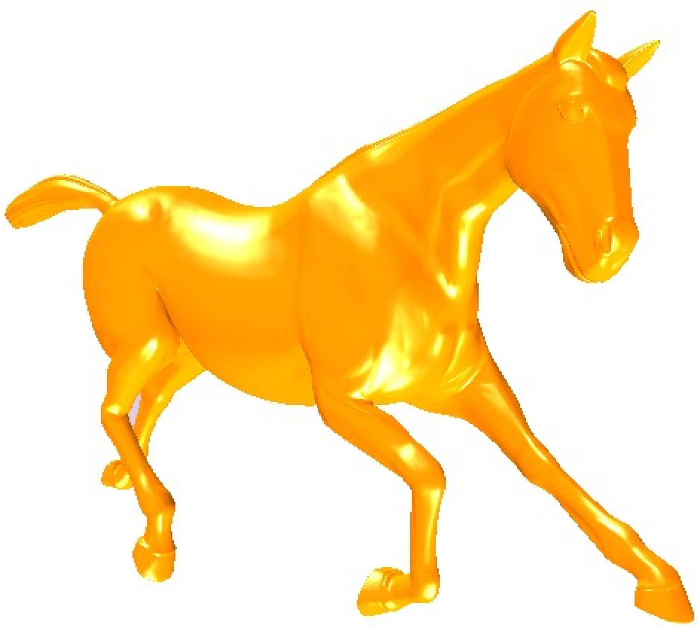
Numerical stability

Covers both polygon form and surface form
(Uniformity)

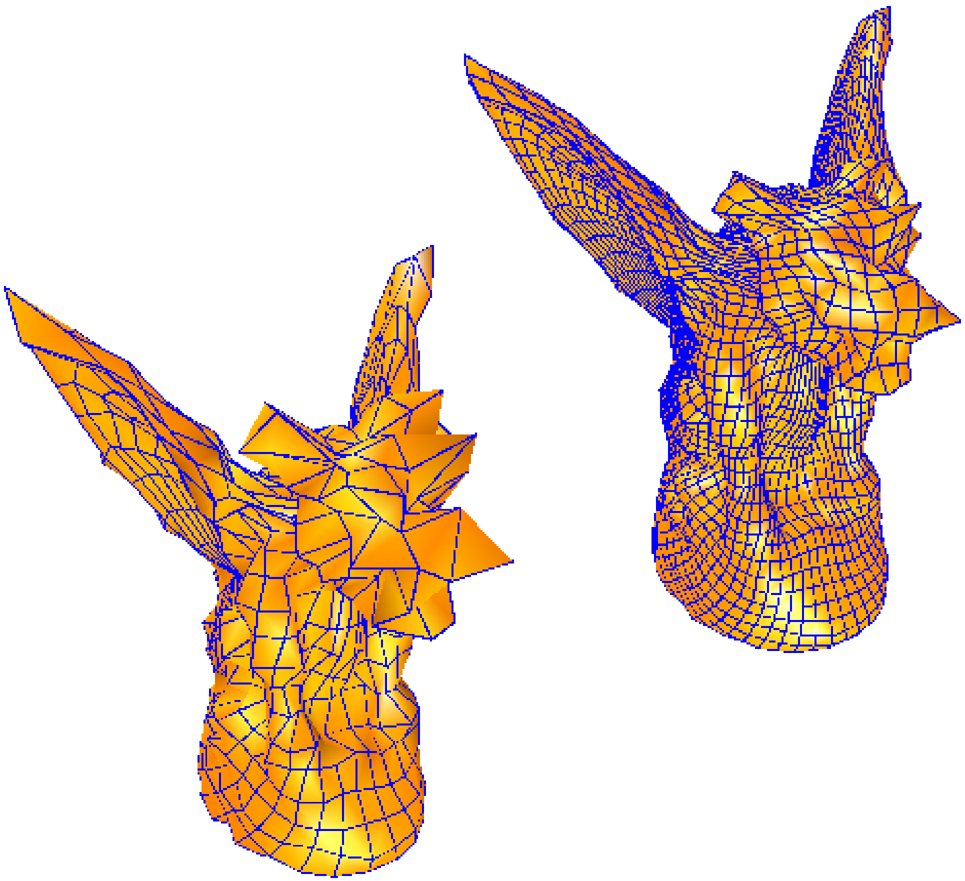
Code Simplicit
y

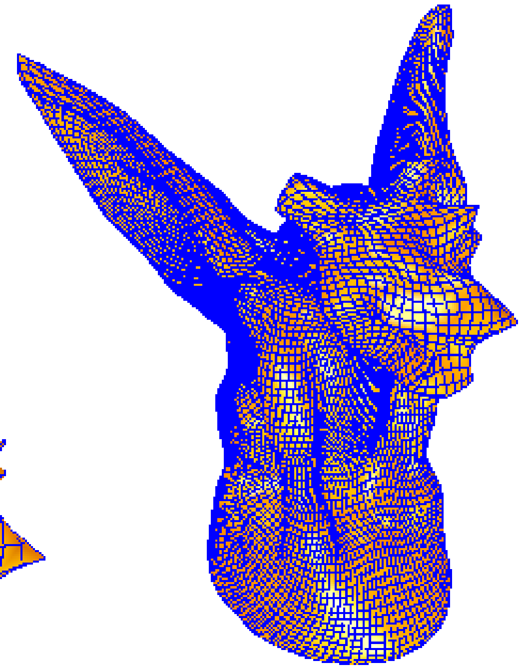


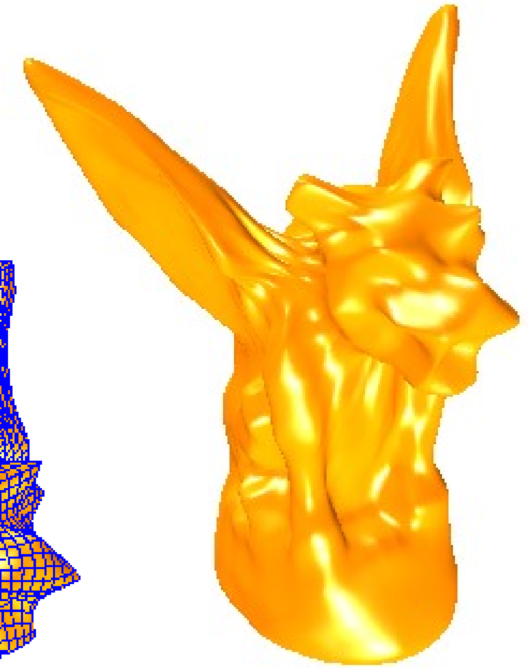
One piece representation™

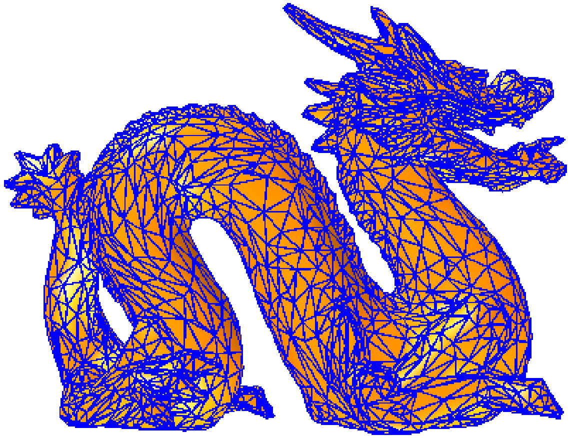


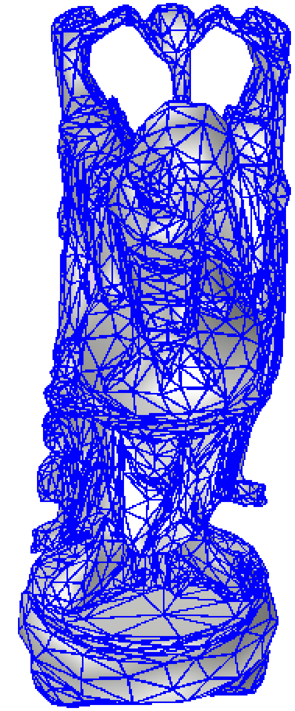
Multi-resolution (Scalability)





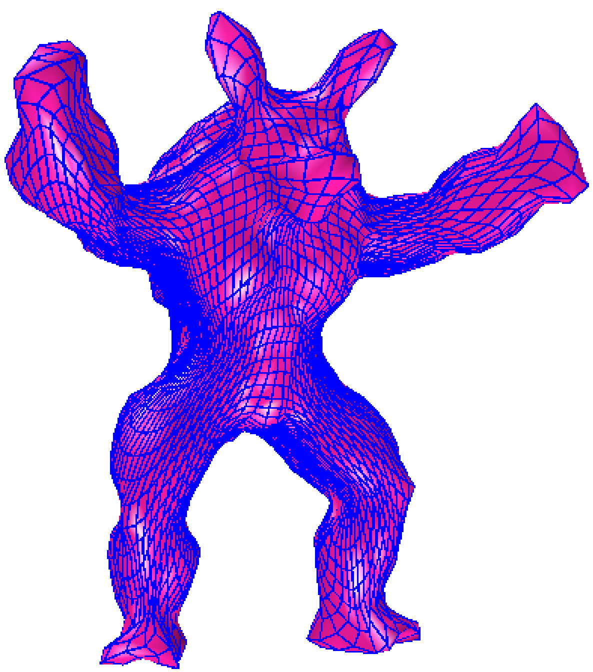






Covers both polygon form and surface form

(Uniformity of representation)



Catmull-Clark

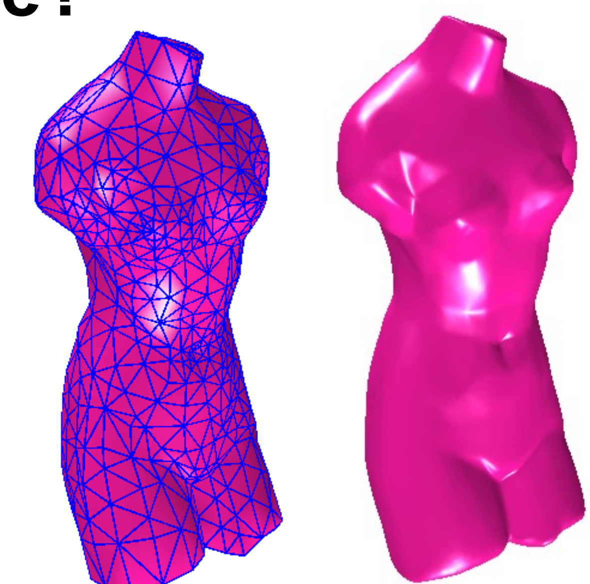
Quadrilateral

Doo-Sabin

So, just what is a subdivision surface?

Triangular

Loop

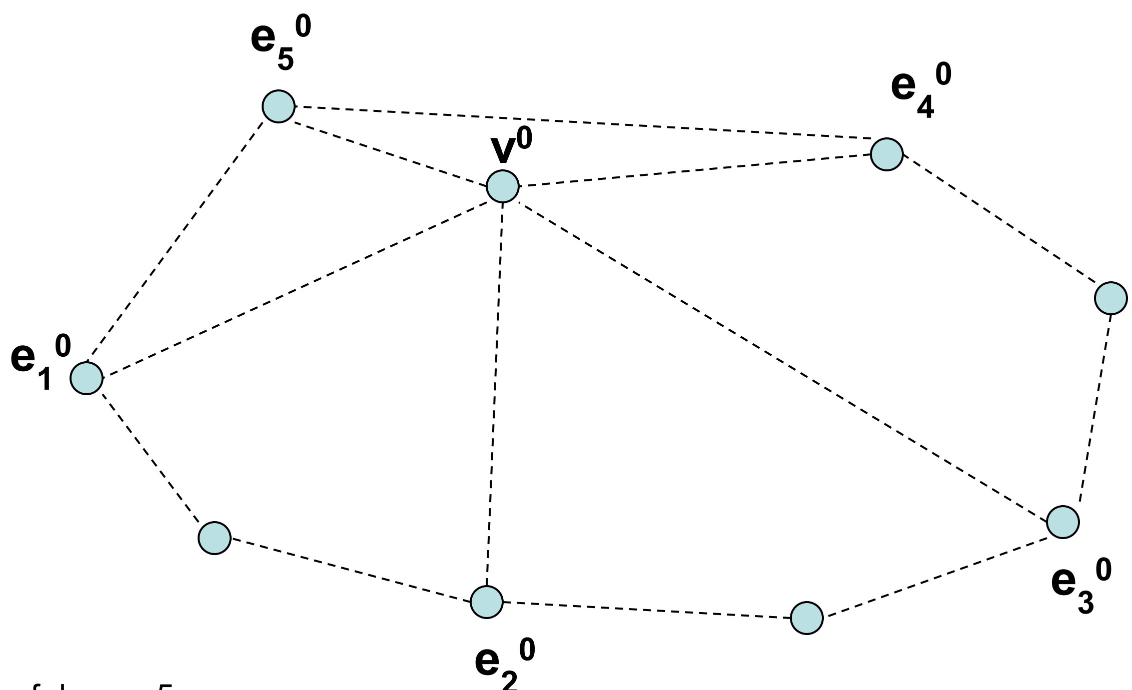


Butterfly

Basic Concept (Catmull-Clark Scheme):

: vertices from mesh M^0

, , : vertices to be generated for M^1

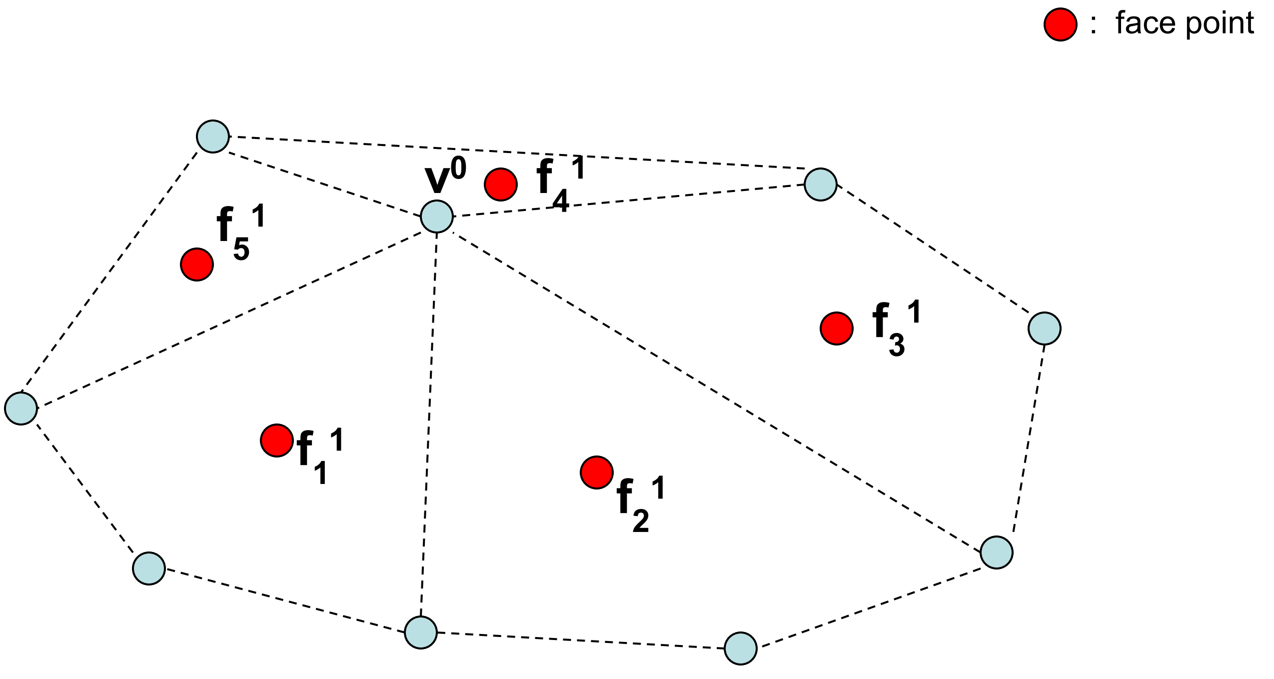


Around a vertex *v* of degree 5

Basic Concept (Catmull-Clark Scheme):

Generating new face points

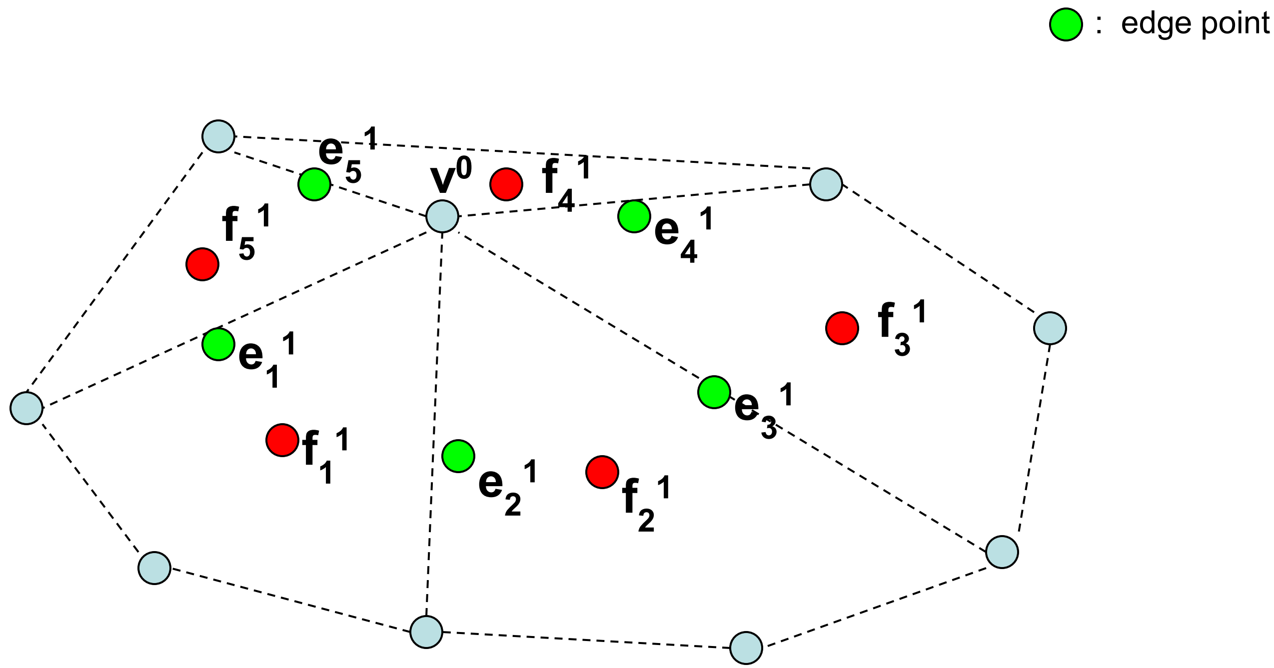
Face point: centroid of each face



Basic Concept (Catmull-Clark Scheme):

Generating new edge points

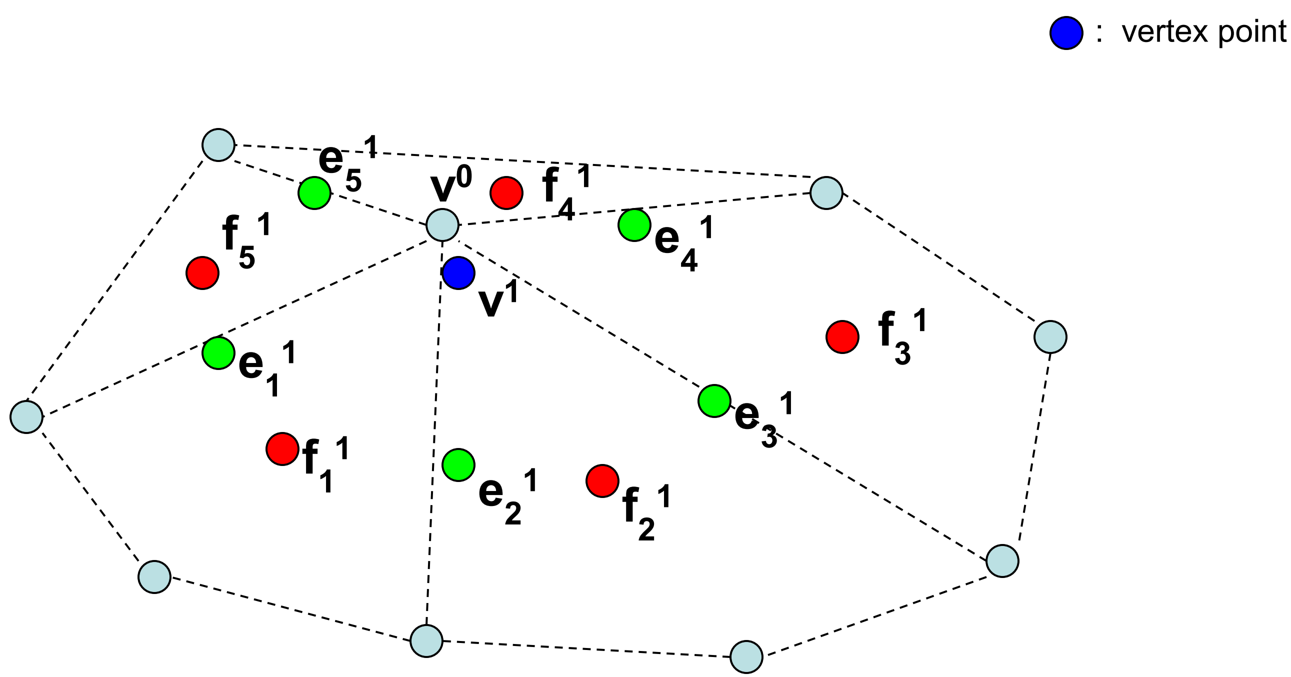
$$e_i^1 = \frac{v^0 + e_i^0 + f_{i-1}^1 + f_i^1}{4}$$



Basic Concept (Catmull-Clark Scheme):

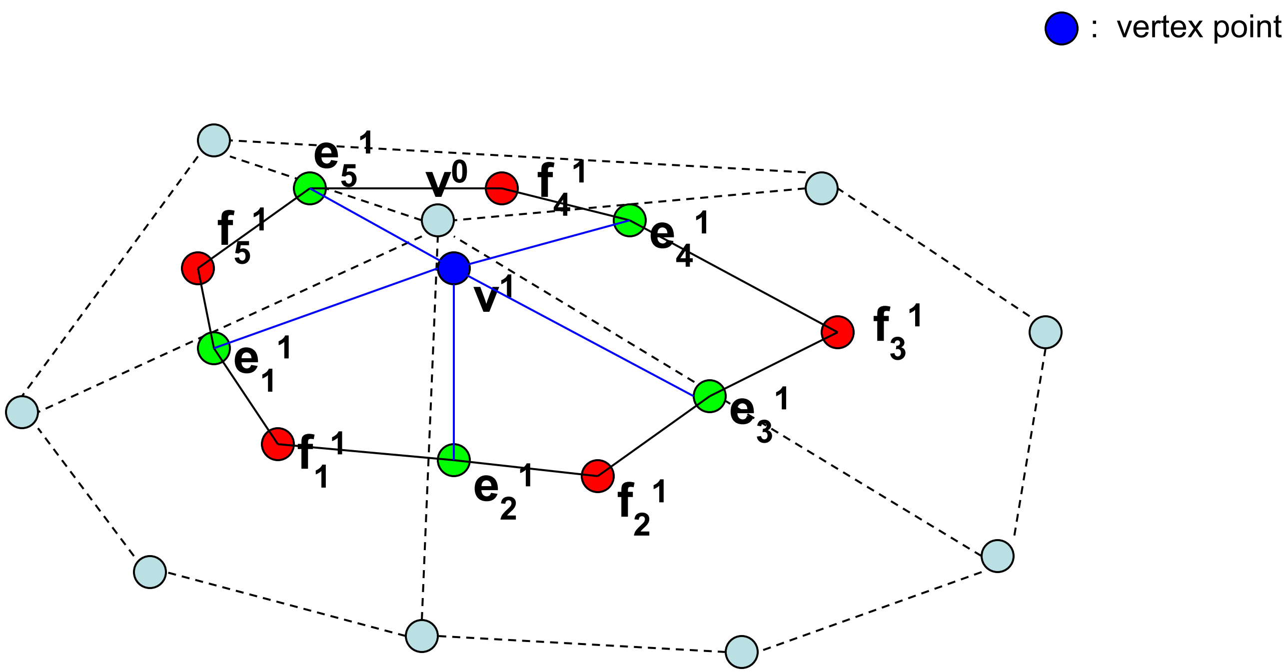
Generating new vertex points

$$v^1 = \frac{n-2}{n} v^0 + \frac{1}{n^2} \sum e_i^0 + \frac{1}{n^2} \sum f_i^1$$

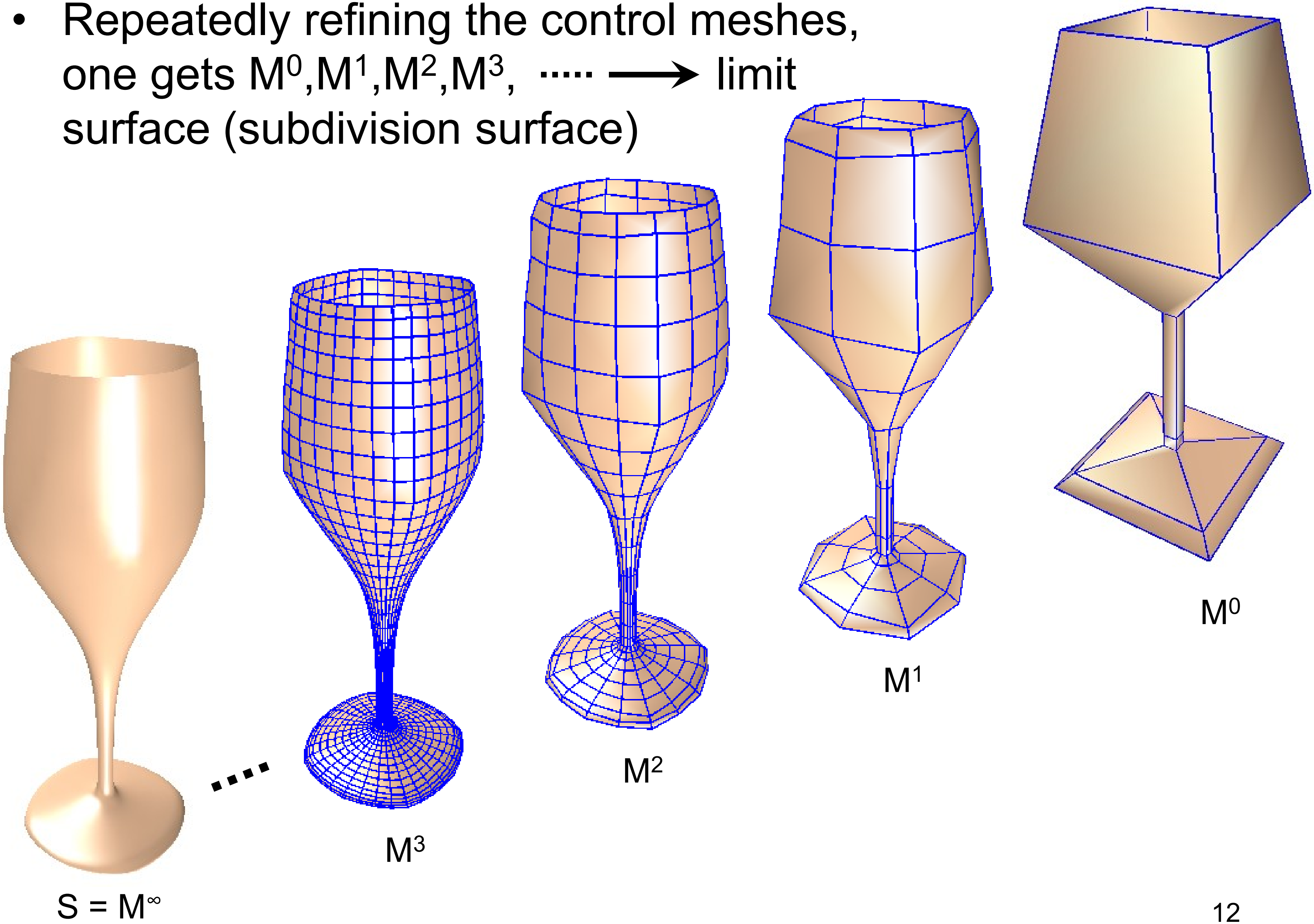


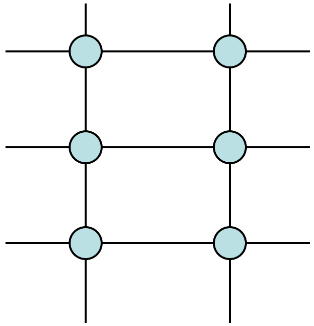
Basic Concept (Catmull-Clark Scheme):

Forming new edges

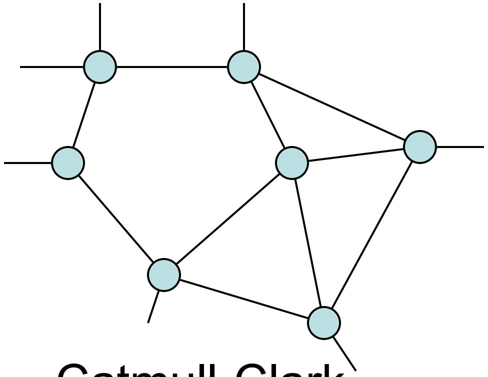


- Repeatedly refining the control meshes, one gets M^0, M^1, M^2, M^3, ····· ⟶ limit surface (subdivision surface)





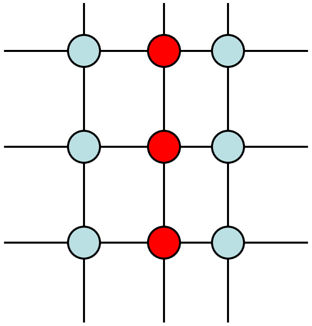
NURBS



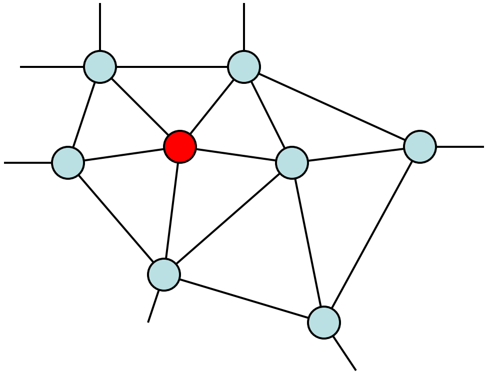
Catmull-Clark

Modeling made much easier. Why?

- No restrictions on the topology of the control points
- Local refinement is possible

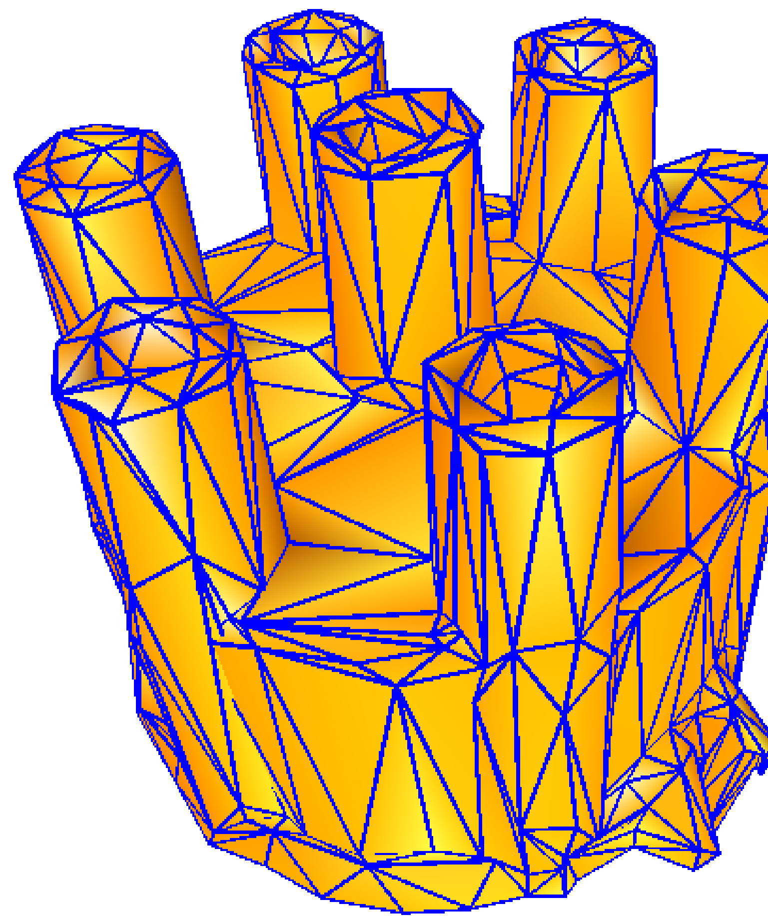


NURBS

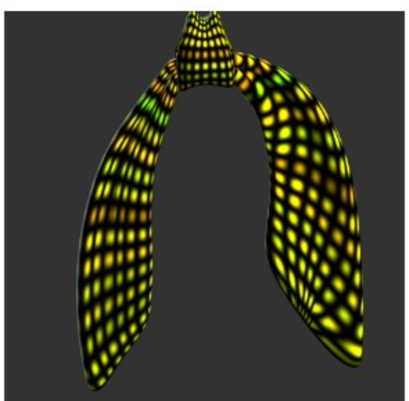


Catmull-Clark

Example of control meshes of Catmull-Clark subdivision surfaces



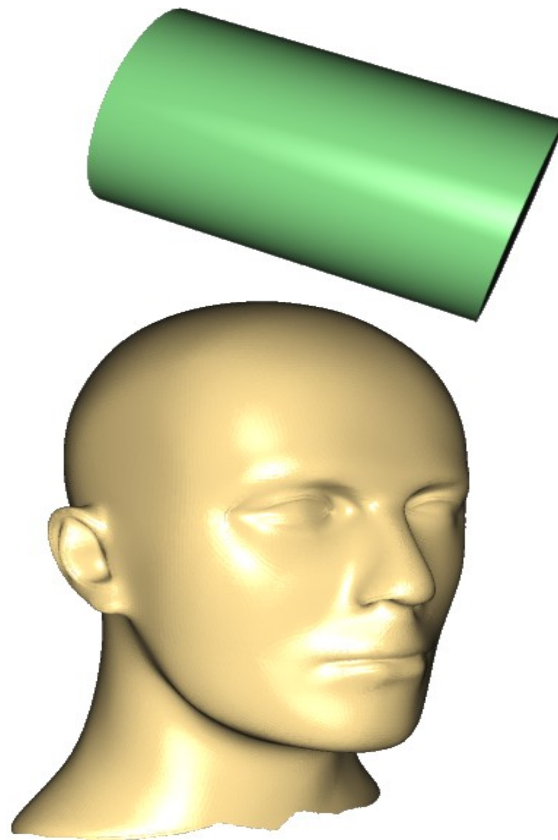

Can model any kind of special features (by modifying the subdivision rules)

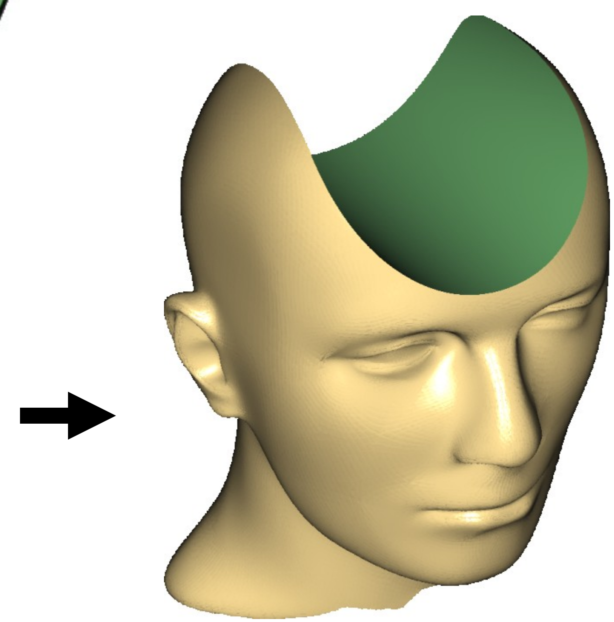

Most importantly, can represent any shape with just one surface

(one piece representation™)

One Piece



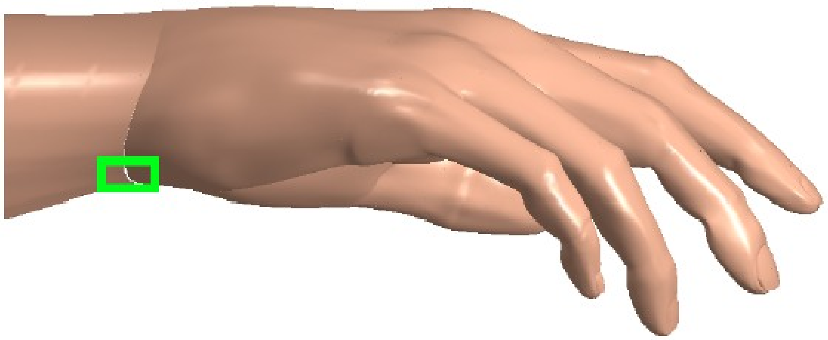
Solid Modeling

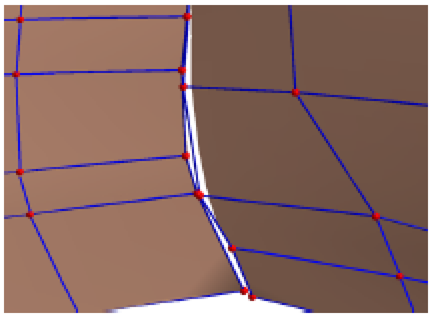


Multi-Piece

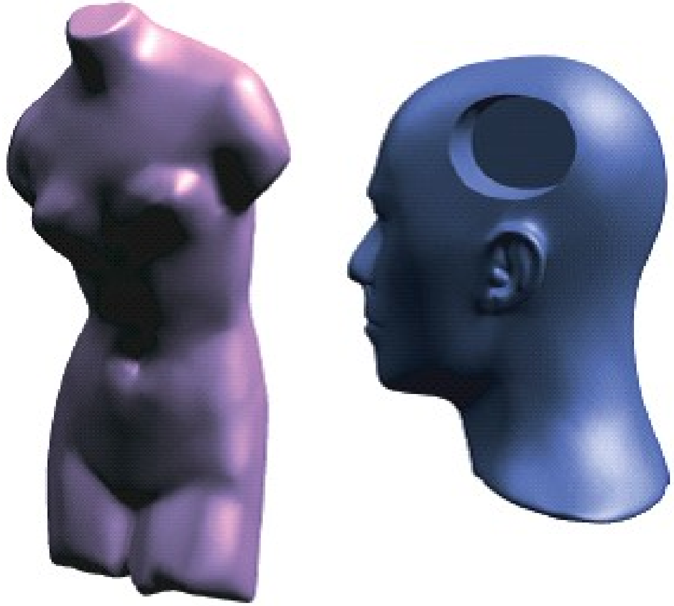
Is One Piece Representation™ Good?

Data Management:	Simpler
Rendering:	More efficient
Machining:	More precise
Animation:	Crack free



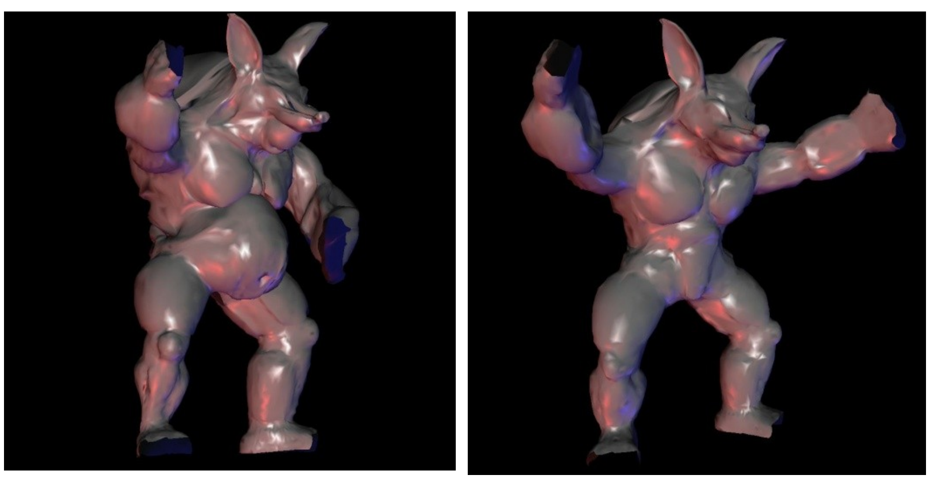


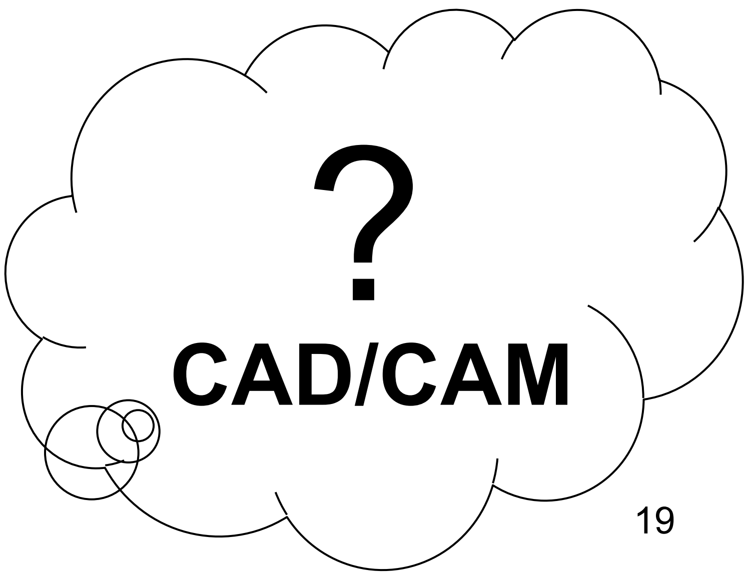
Does this mean
the solid modeling area
is no longer needed?



What is subdivision based representation?

Subdivision Surfaces





What is missing?

1. No parameterization
2. No error control
3. No adaptive tessellation

- Without error control

    No CAD/CAM applications

- Without parameterization

    Difficult to perform picking, rendering, texture mapping

- Without adaptive tessellation

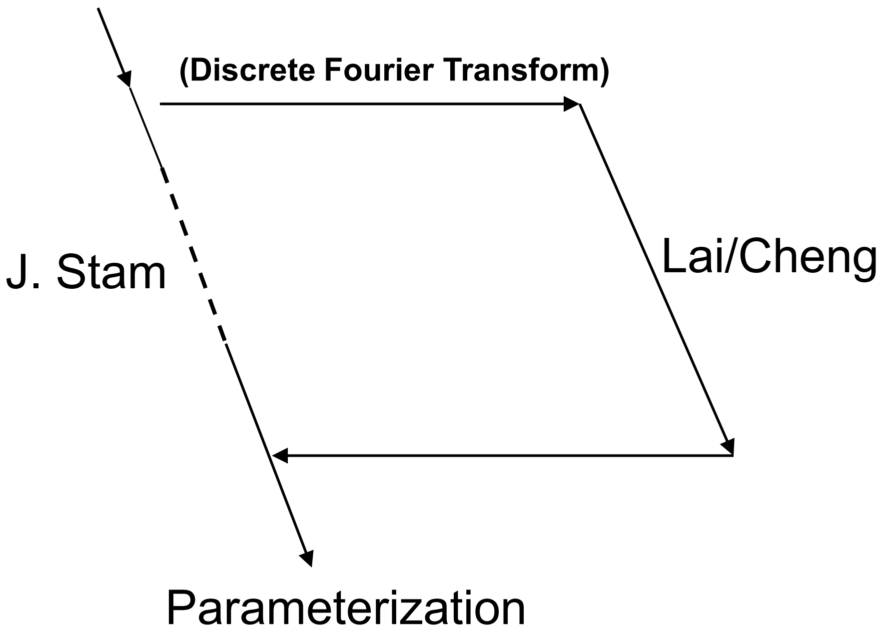
    Too expensive to use

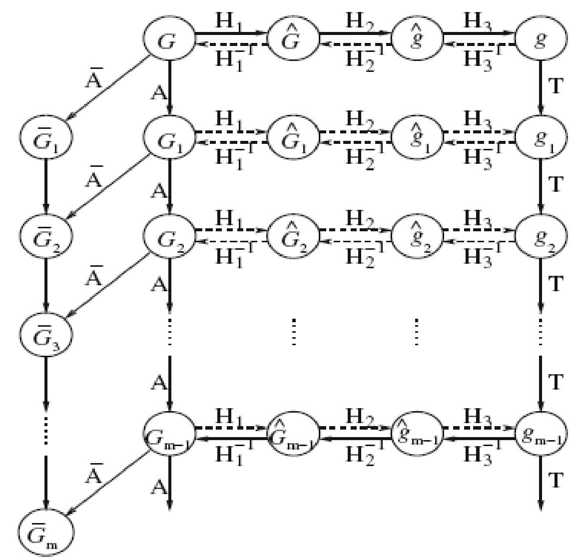
A major breakthrough occurred in 1998

- Jos Stam
- Parameterization of Catmull-Clark Subdivision Surfaces
- 1998

Work on Subdivision Surface Parameterization

1. J. Stam (1998)
2. D. Zorin, D. Kristjansson (2002)
3. S. Lai, F. Cheng (2005)



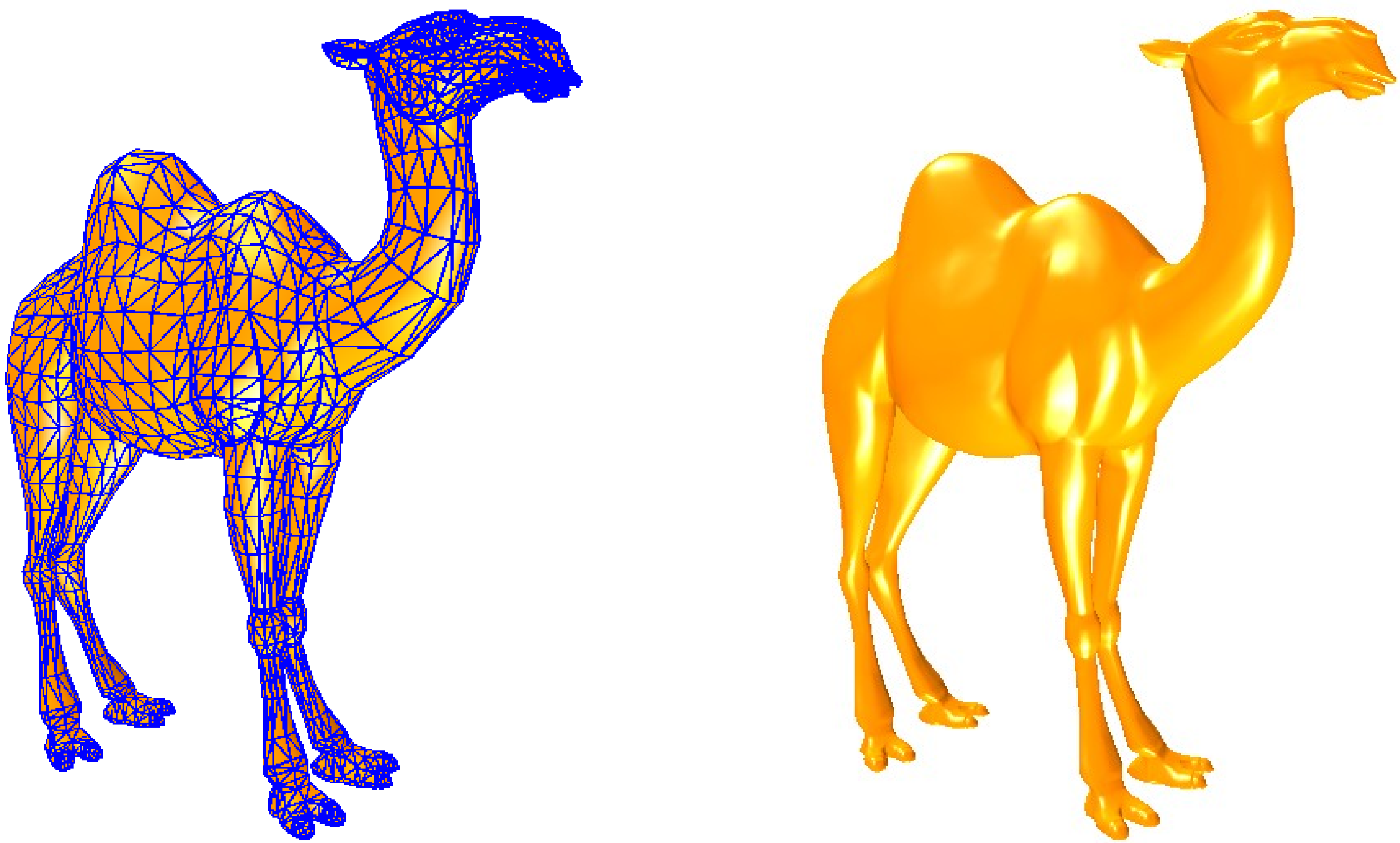


The Extended Subdivision Diagram

Applications of the new parameterization technique

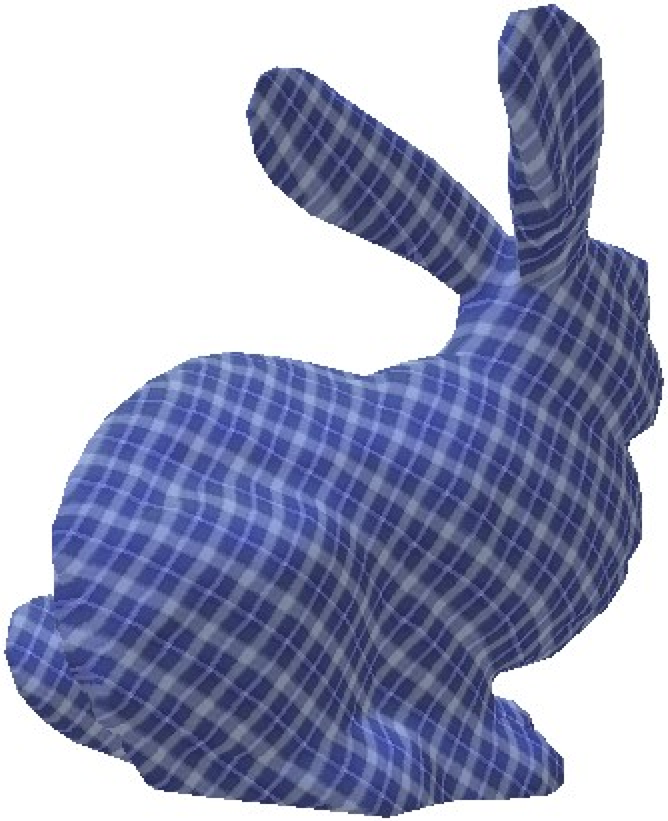
- Surface Evaluation
- Texture Mapping
- Boolean Operations
- Surface Trimming
- Adaptive Tessellation
- Animation

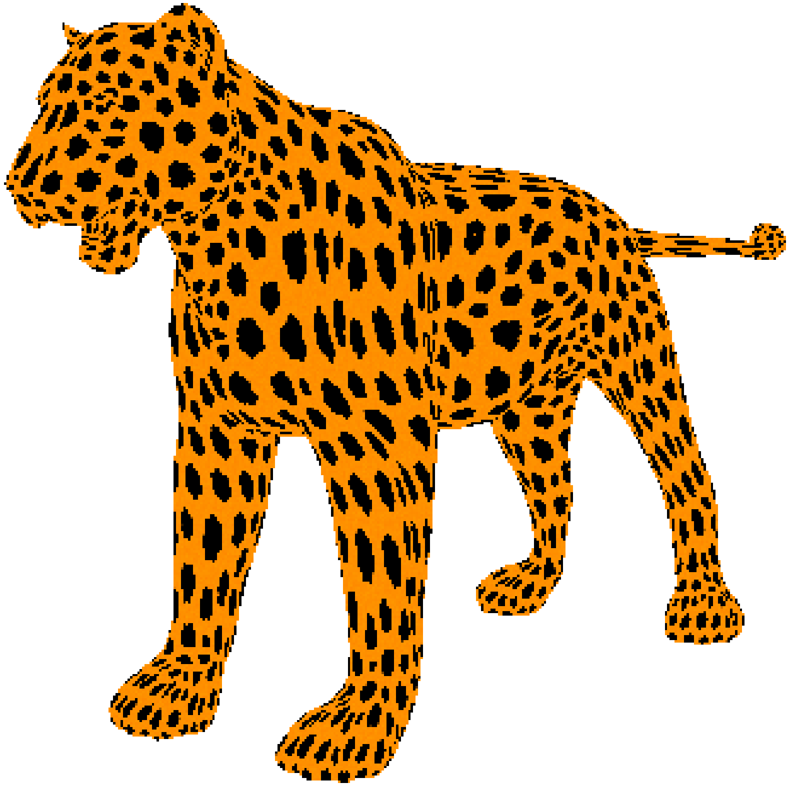
Surface Evaluation



Fast, Exact Rendering

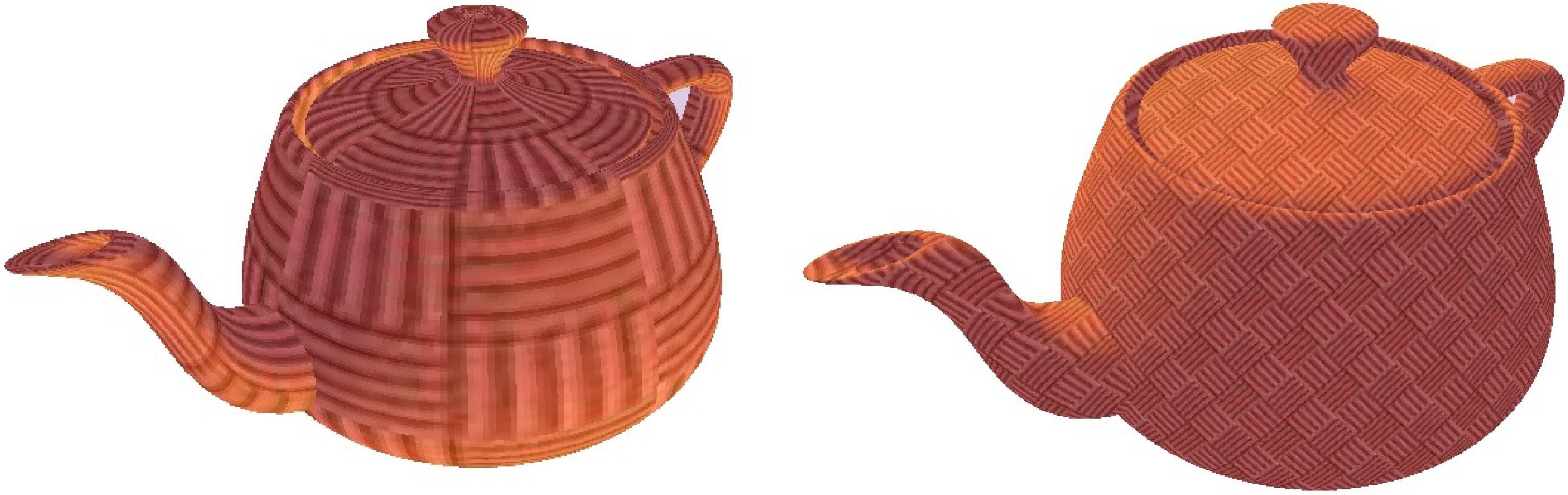
Texture Mapping[1]





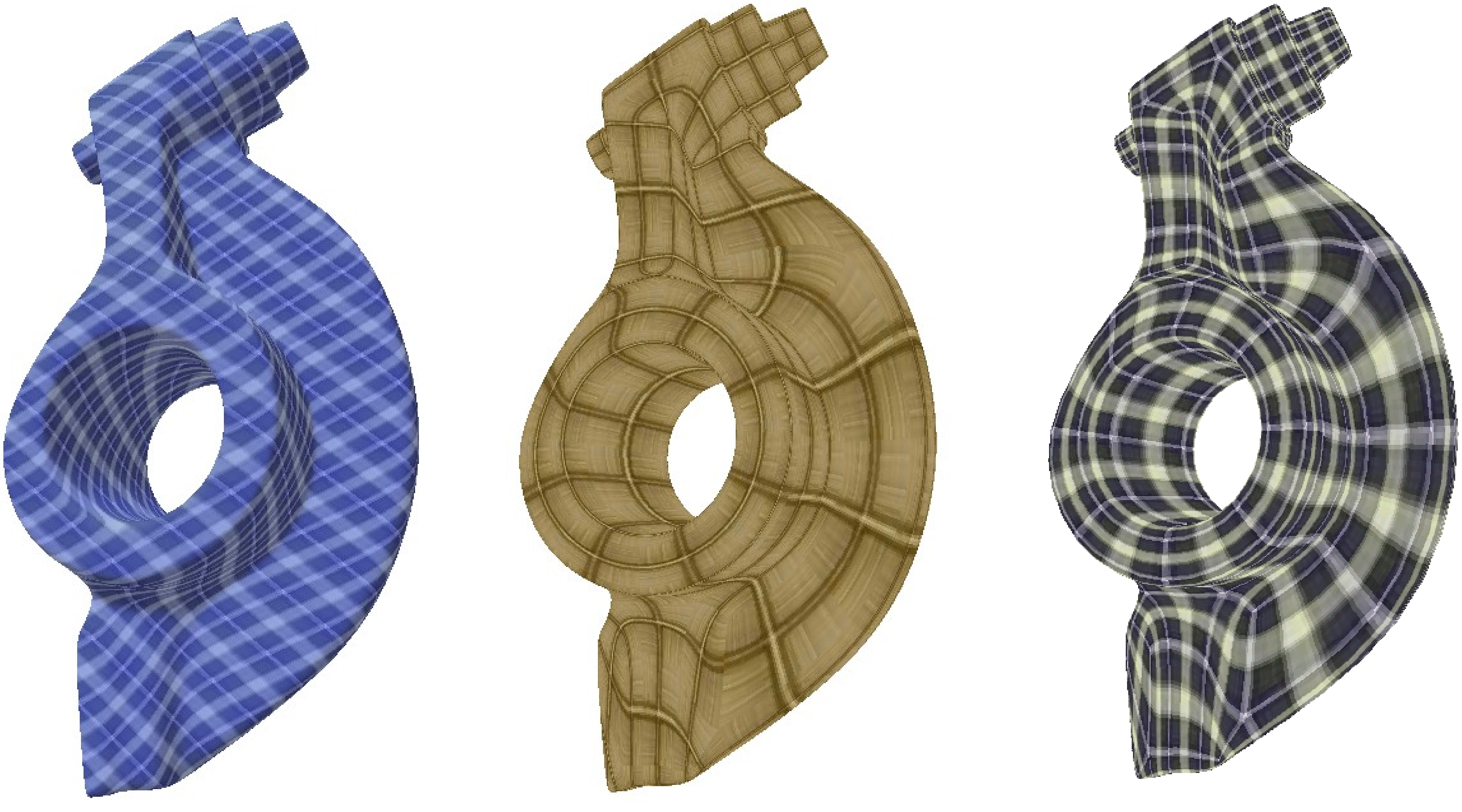
1: Lai and Cheng, 2005

Texture Mapping[1]



1: Lai and Cheng, 2005

Texture Mapping[1]

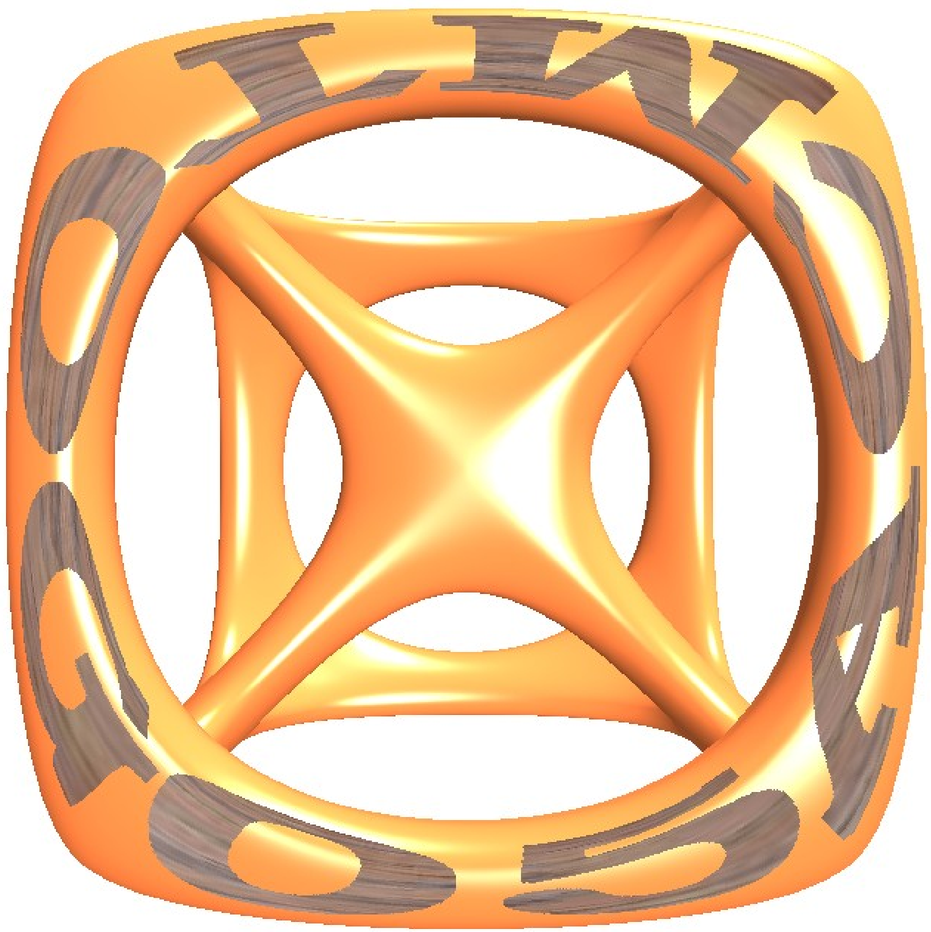


1: Lai and Cheng, 2005

Boolean Operations[2]

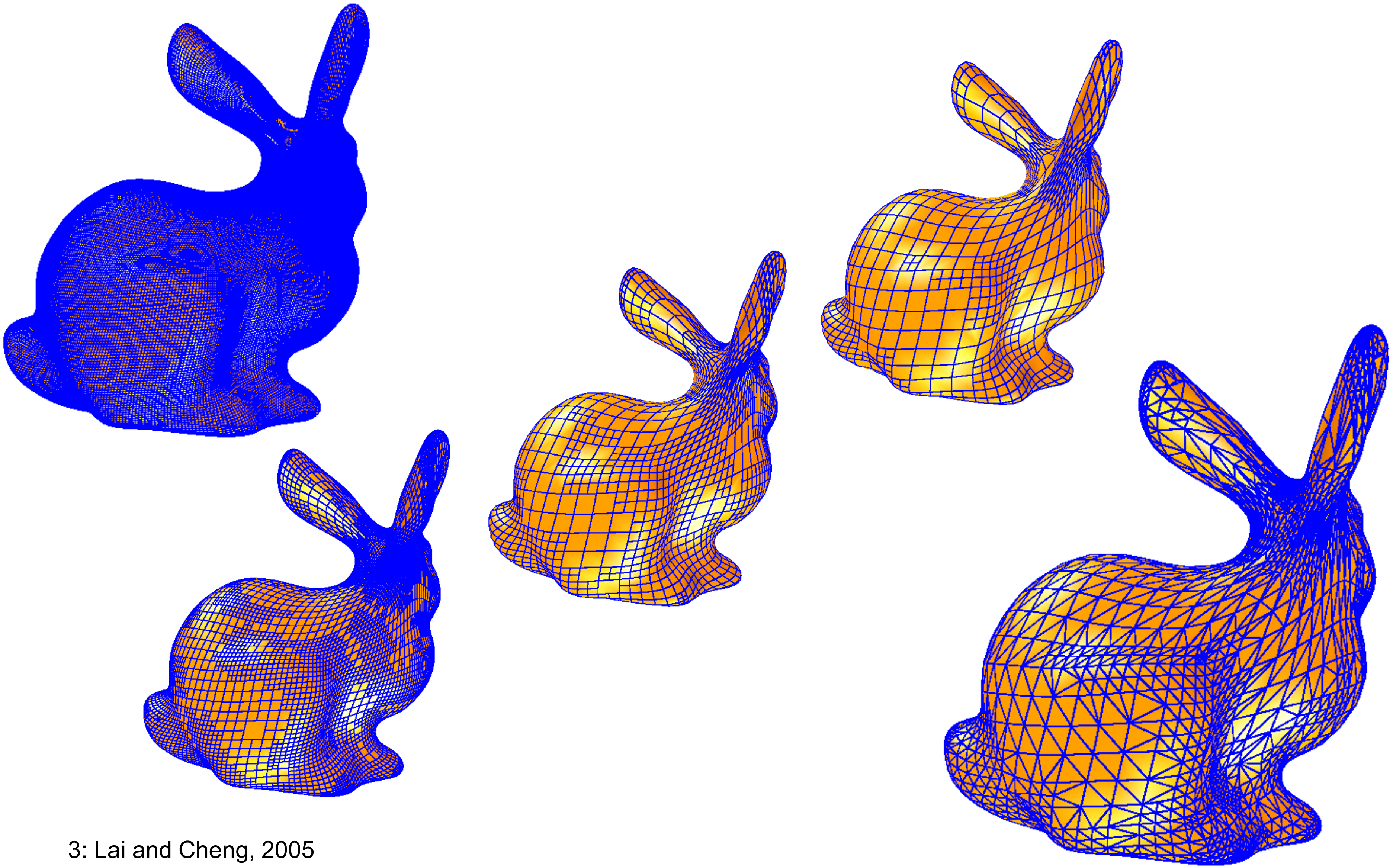
2: Lai and Cheng, 2005

Surface Trimming[2]



2: Lai and Cheng, 2005

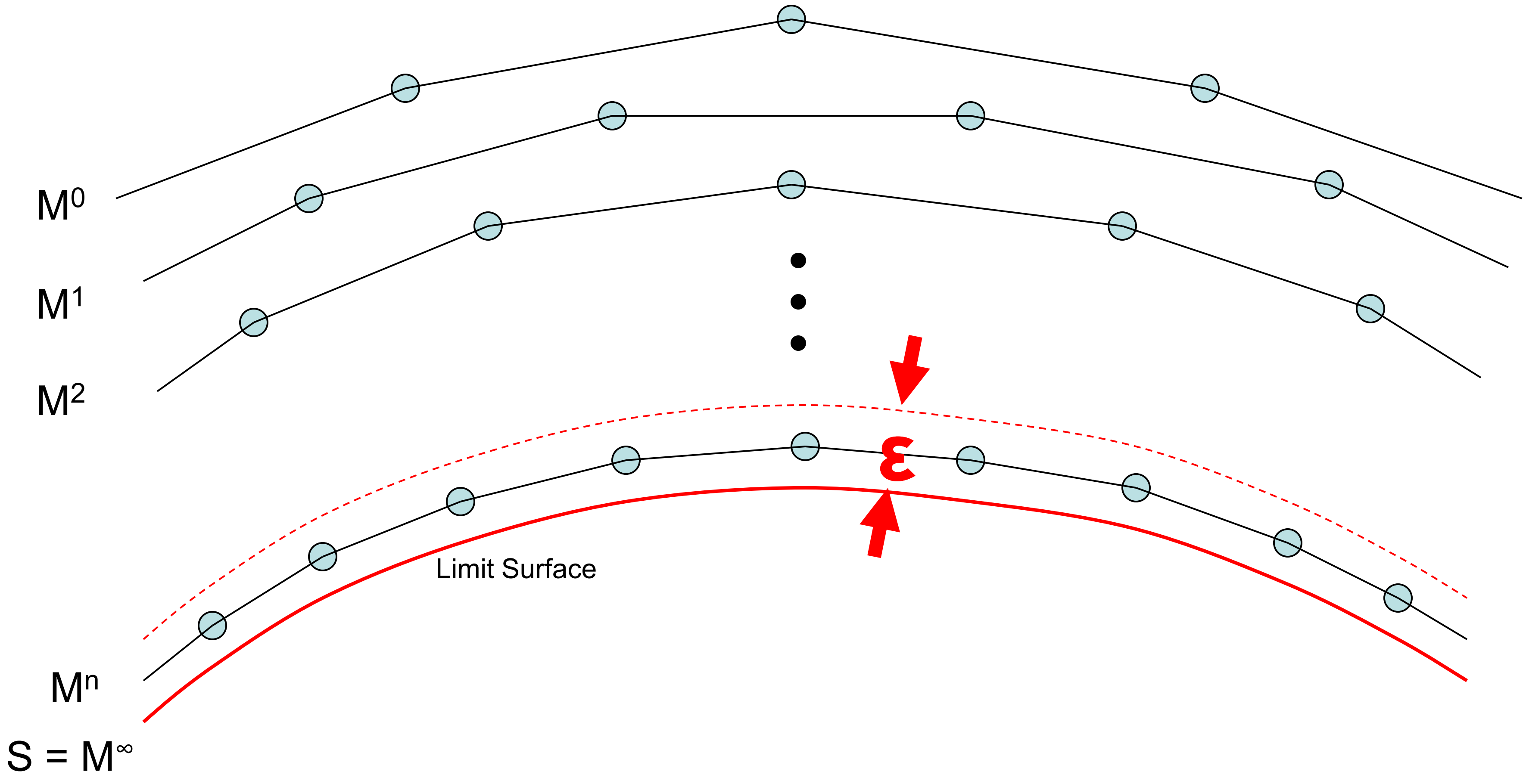
Adaptive Tessellation[3]



3: Lai and Cheng, 2005

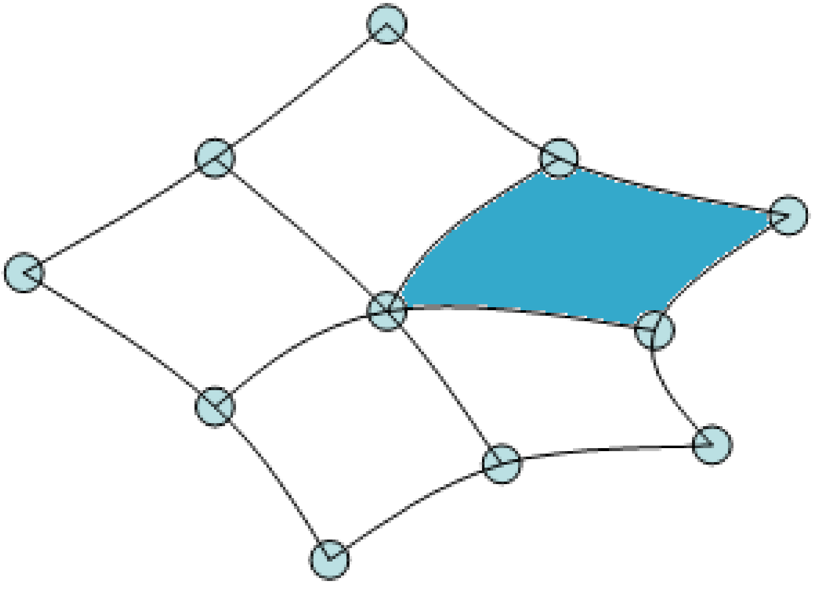
What is error control?

Error Control: Given $\varepsilon > 0$, when would $\| M^n - S \| < \varepsilon$?



Cross-Sectional View

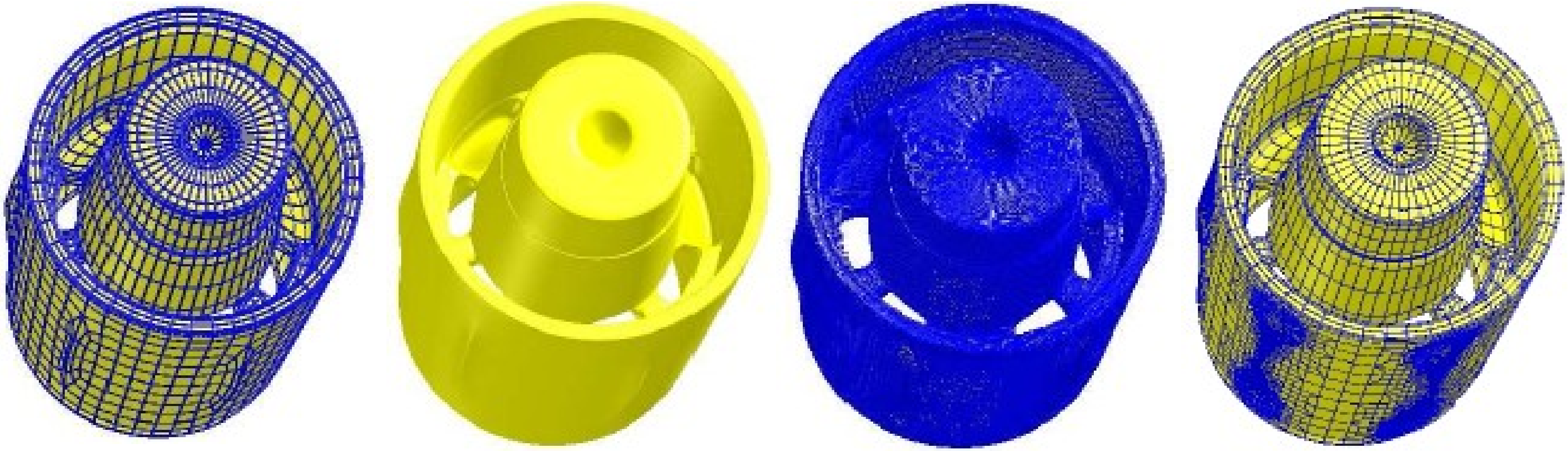
What metric should we use to assess $\|M^n - S\|$ for an extra-ordinary patch?



A solution is finally available…

- F. Cheng, G. Chen, J. Yong
- Subdivision Depth Computation for Catmull-Clark Subdivision Surfaces
- 2005

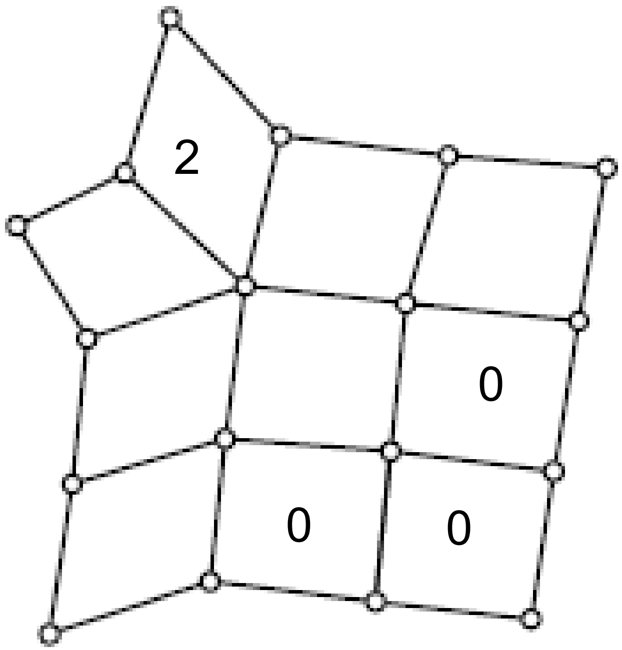
This work is also important for adaptive subdivision[5].

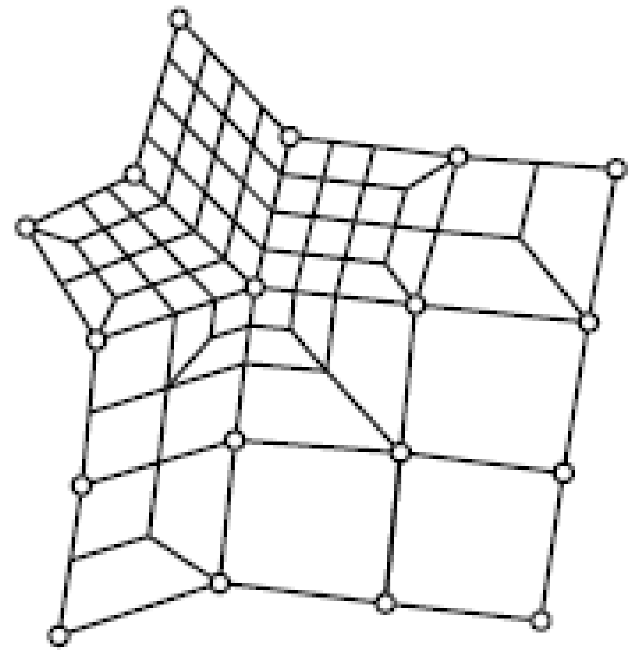


Control Mesh Limit Surface Uniform Subdivision Adaptive Subdivision

5: J. Yong, F. Cheng, (2004)

Basic Idea: Use unbalanced subdivision[6] to provide smooth transition between areas with different densities





6: F. Cheng, J. Jaromczyk et al (1989)

Adaptive subdivision:

input: a piecewise surface P and a subdivision level assignment S

output: a triangular linear approximaiton P** of P

Three phases:

Phase 1: define a label for each vertex of P

Phase 2: generate a gradrilateral subdivision mesh P* of P

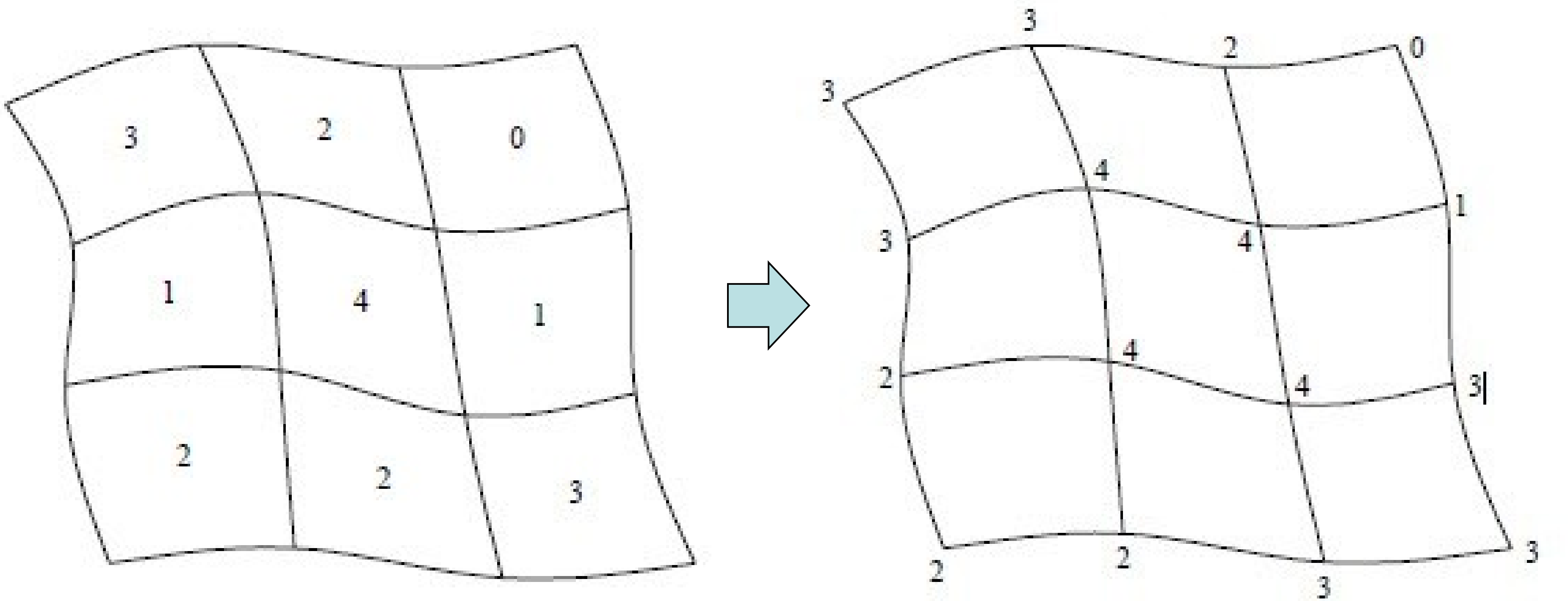
Phase 3: convert P* to a triangular linear approximation P** of P

Phase 1:

/* F $\equiv$ { f | f is a patch of P } */

<u>for</u> each vertex v of P <u>do</u>

L (v) := max({1} $\cup$ { S(f) | f $\in$ F, v is a vertex of f })



Phase 2:

1. <u>for</u> each vertex v of P <u>do</u>
   LABEL(v) := L(v);

2. <u>for</u> each patch f of P <u>do</u>
   Subdivide(f);

Subdivide(f : quadrilateral surface patch);

if (LABEL(v) > 0 for more than one vertex of f) then

balanced_sub(f, f_1, f_2, f_3, f_4);

for i :=1 to 4 do

subdivide(f_i);

else if (LABEL(v) > 0 for only one vertex of f) then

unbalanced_sub(f, f_1, f_2, f_3);

for i :=1 to 3 do

subdivide(f_i);

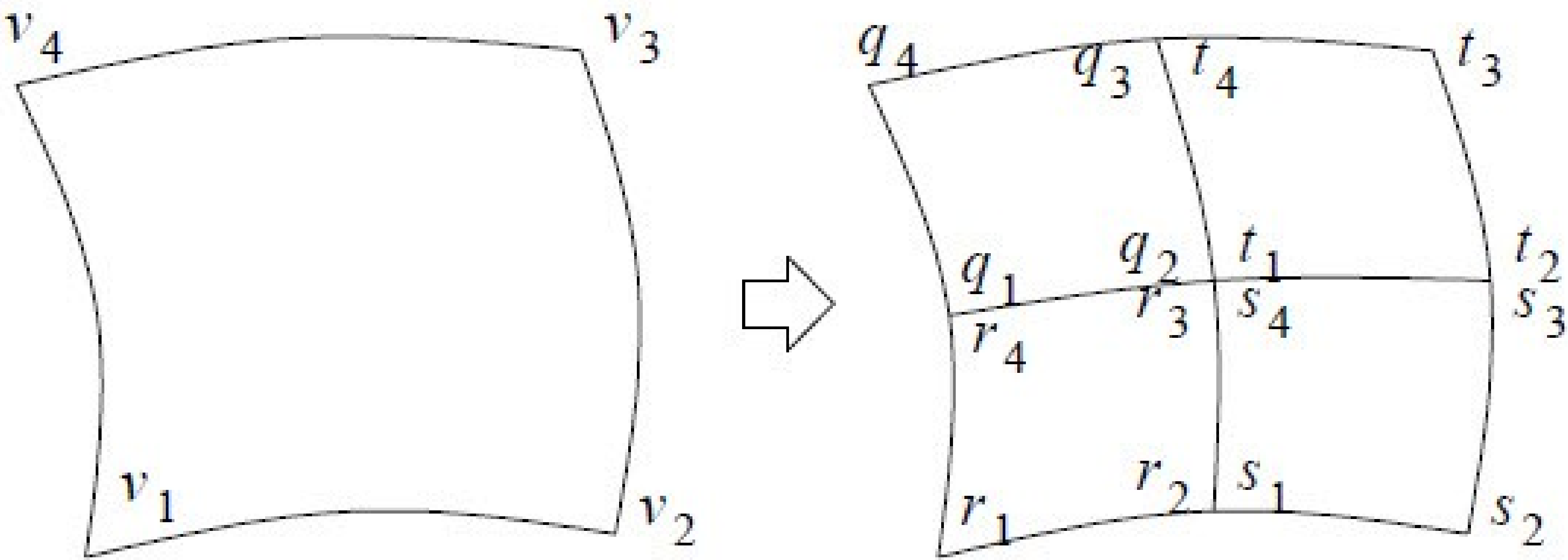
balanced_sub(f):

Perform mid-point subdibision on f to get four new subpatches: $r_1r_2r_3r_4$, $s_1s_2s_3s_4$, $t_1t_2t_3t_4$, $q_1q_2q_3q_4$, and assign new labels as follows:

$$LABEL(r_1) = \max\{0, LABEL(v_1) - 1\}$$
$$LABEL(s_2) = \max\{0, LABEL(v_2) - 1\}$$
$$LABEL(t_3) = \max\{0, LABEL(v_3) - 1\}$$
$$LABEL(q_4) = \max\{0, LABEL(v_4) - 1\}$$
$$LABEL(r_2) = LABEL(s_1) = \min\{LABEL(r_1), LABEL(s_2)\}$$
$$LABEL(s_3) = LABEL(t_2) = \min\{LABEL(s_2), LABEL(t_3)\}$$
$$LABEL(t_4) = LABEL(q_3) = \min\{LABEL(t_3), LABEL(q_4)\}$$
$$LABEL(q_1) = LABEL(r_4) = \min\{LABEL(q_4), LABEL(r_1)\}$$

$$LABEL(r_3) = LABEL(s_4) = LABEL(t_1) = LABEL(q_2)$$

$$= \begin{cases} 0, & \text{if } r_2,\ s_3,\ t_4,\ and\ q_1\ are\ assigned\ zero\ label \\ \min\{LABEL(v) \mid v \in \{r_2, t_3, t_4, q_1\}, LABEL(v) > 0\}, & otherwise \end{cases}$$



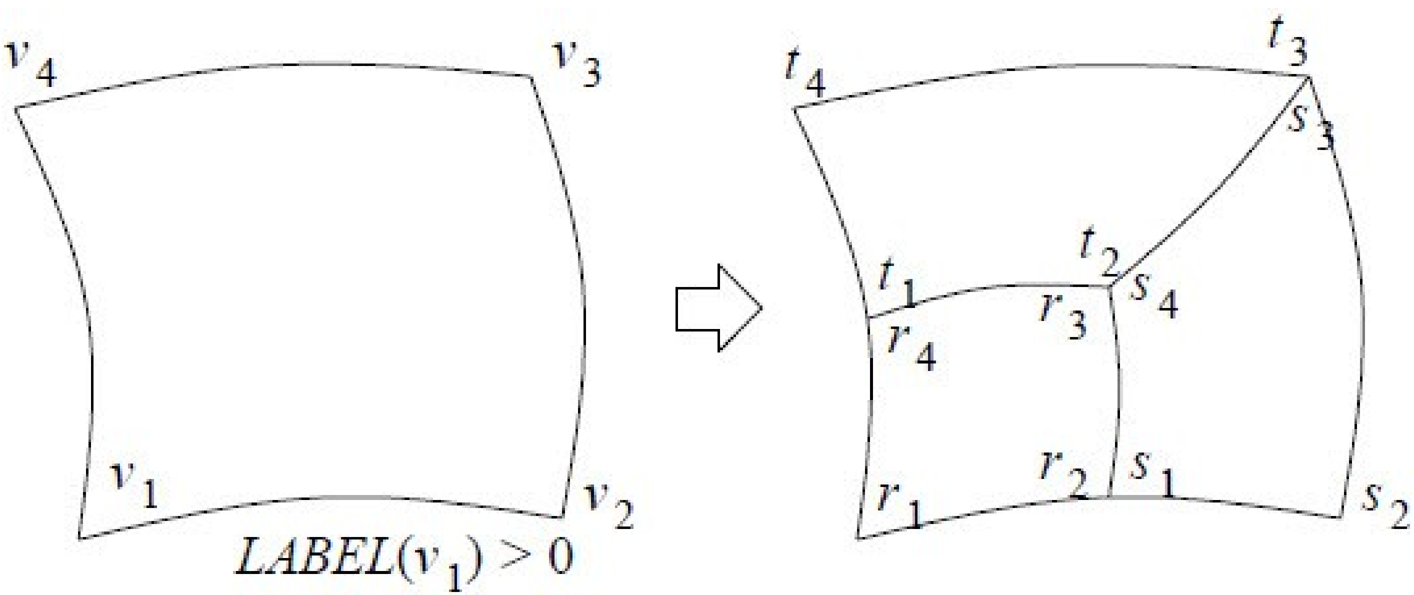
unbalanced_sub(f):

If $LABEL(v_1) > 0$, subdivide f as above to get three new subpatches: $r_1r_2r_3r_4$, $s_1s_2s_3s_4$, $t_1t_2t_3t_4$, and assign new labels as follows:

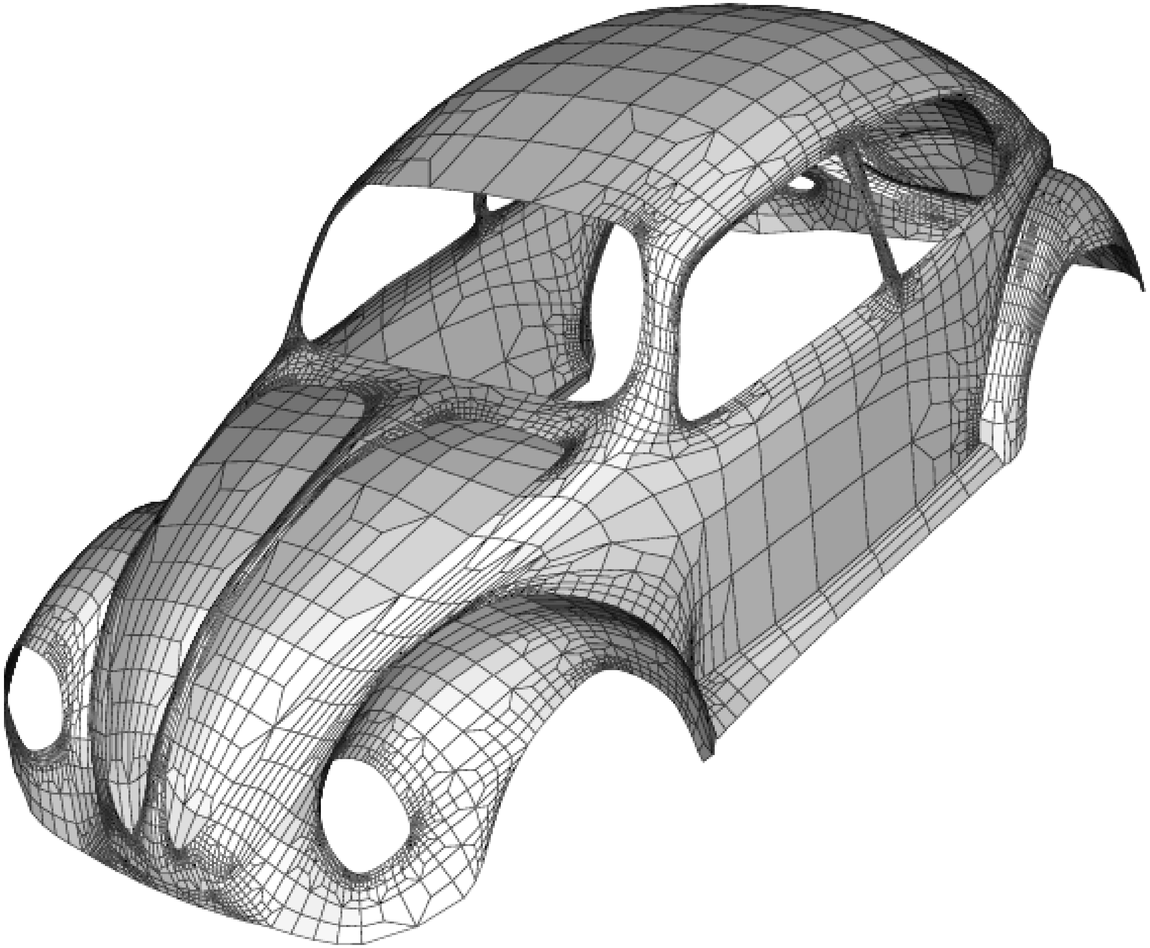
$$LABEL(r_1) = LABEL(v_1) - 1$$

$$LABEL(r_i) = 0, \ \ i = 2,3,4; \qquad LABEL(s_i) = 0, \ \ i = 1,2,3,4$$

$$LABEL(t_i) = 0, \ \ i = 1,2,3,4.$$



Example of adaptive subdivision



Significant savings

Subdivision surfaces have already been used in

- Pixar's Renderman
- Alias|Wavefront's Maya
- Nichimen's Mirai
- Newtek's Lightwave 3D

⋮

Question:

"Is subdivision the representation scheme for future visualization & animation applications?"

The  End

Acknowledgement:

- Research work presented here is supported by NSF (DMS-0310645, DMI-0422126).
- Some datasets are taken from P. Schroeder, D. Zorin, L. Kobbelt, H. Hoppe.