7.4 Rigid body simulation*

Examples - a uniform force field

- a uniform force field has no effect on angular momentum of a body
- a uniform force field can be treated as a single force acting on the rigid body at its mass center

e.g., a gravitational field exerting a force $m_i g$ on each particle of a rigid body, g: downward pointing vector

net force: $F_g = \sum m_i g = Mg$

acceleration of mass center: $Mg / g = g$

net torque:

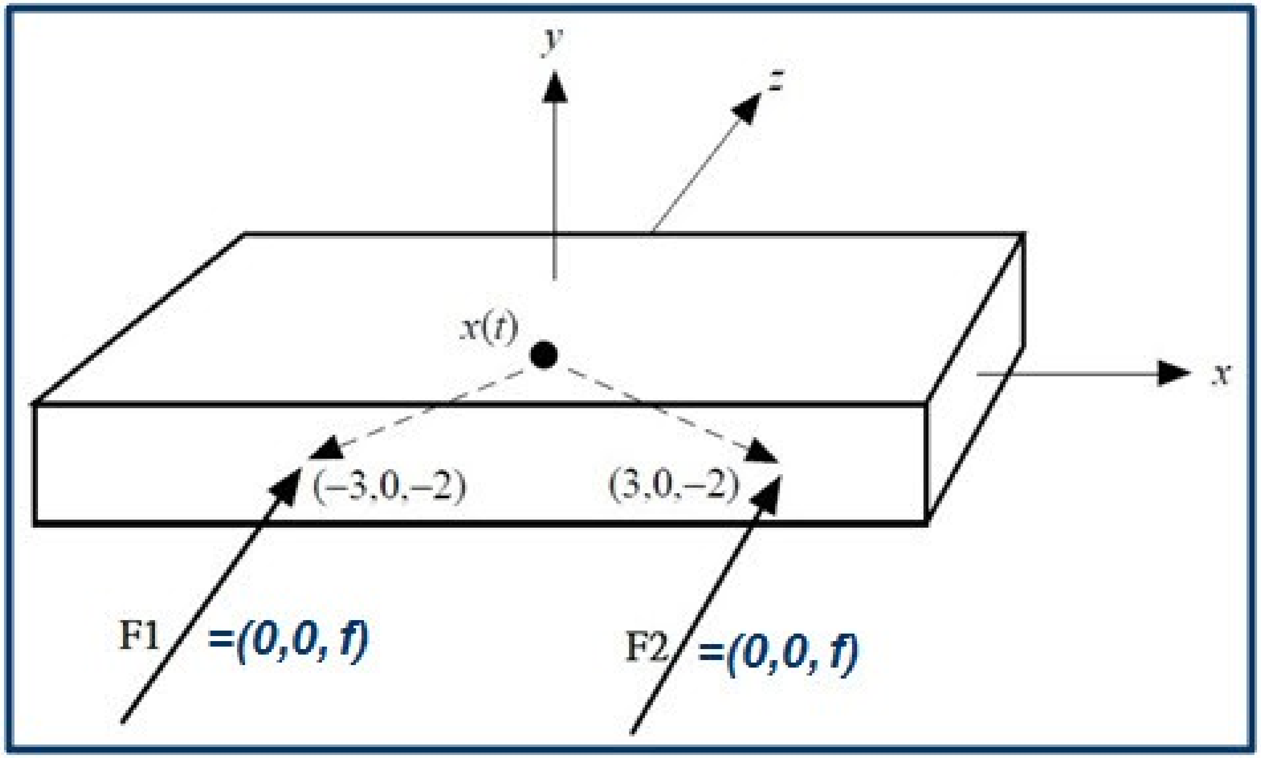
$$\sum (r_i(t) - x(t)) \times m_i g = \left(\sum m_i (r_i(t) - x(t)) \right) \times g = 0$$

* Some materials used here are taken from David Baraff's notes: **Physically Based Modeling - *Rigid Body Simulation***

Examples - rotation free movement of a body

- 2 equal forces acted on two opposite points of the mass center of a body would cause the body to accelerate linearly, without accelerating angularly

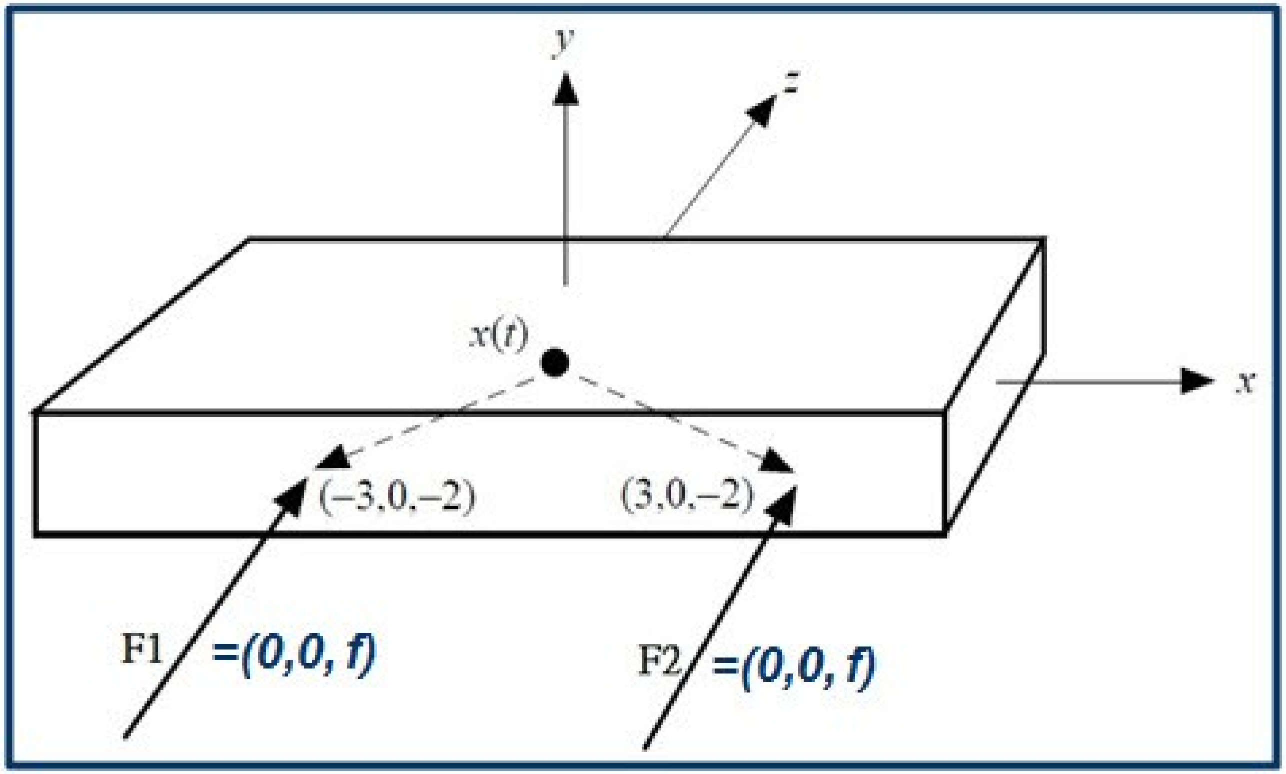
Consider the case shown below:



net force: ***(0,0,2f)***

acceleration of mass center: ***2f/M***

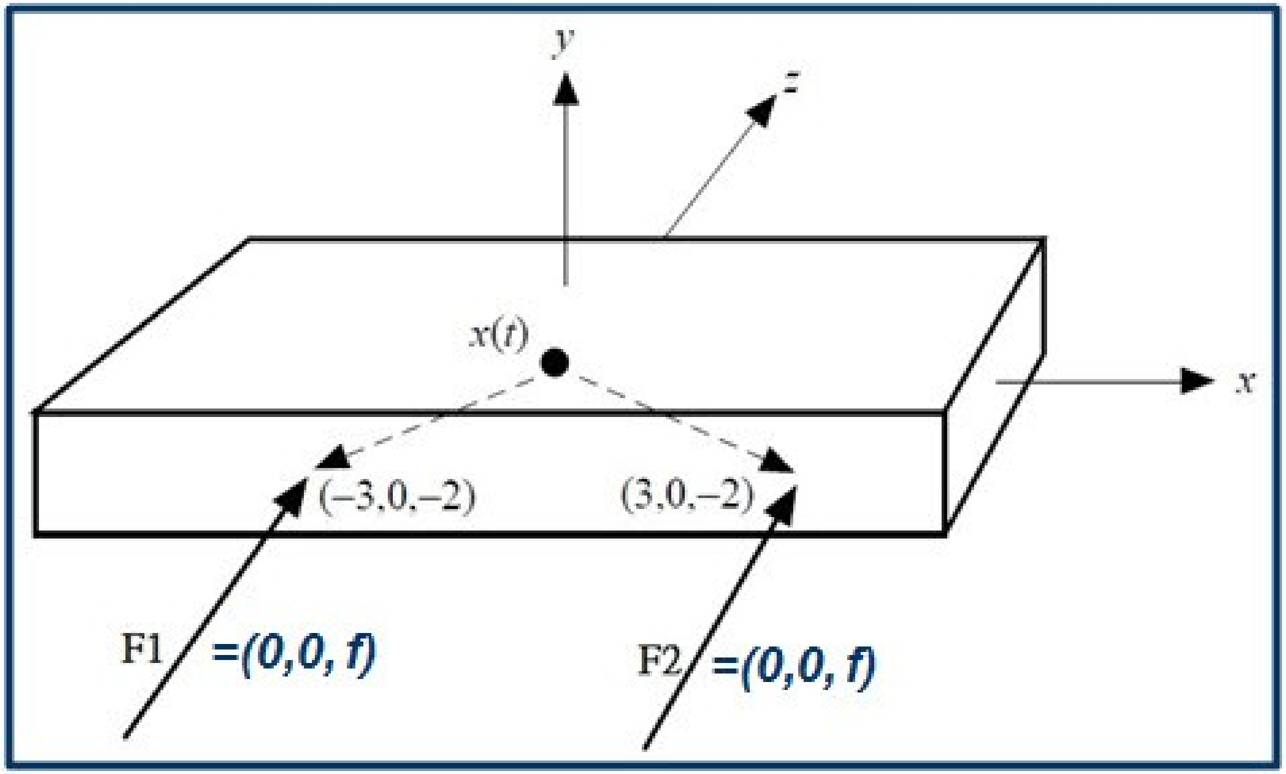
Examples - rotation free movement of a body



torque due to *F1*:

$$\left(\left(x(t) + \begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix}\right) - x(t)\right) \times F1 = \begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix} \times F1$$

Examples - rotation free movement of a body



torque due to *F2*:

$$((x(t) + \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}) - x(t)) \times F2 = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} \times F2$$

Examples - rotation free movement of a body

total **torque:**

$$\tau = \begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix} \times F1 + \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} \times F2$$

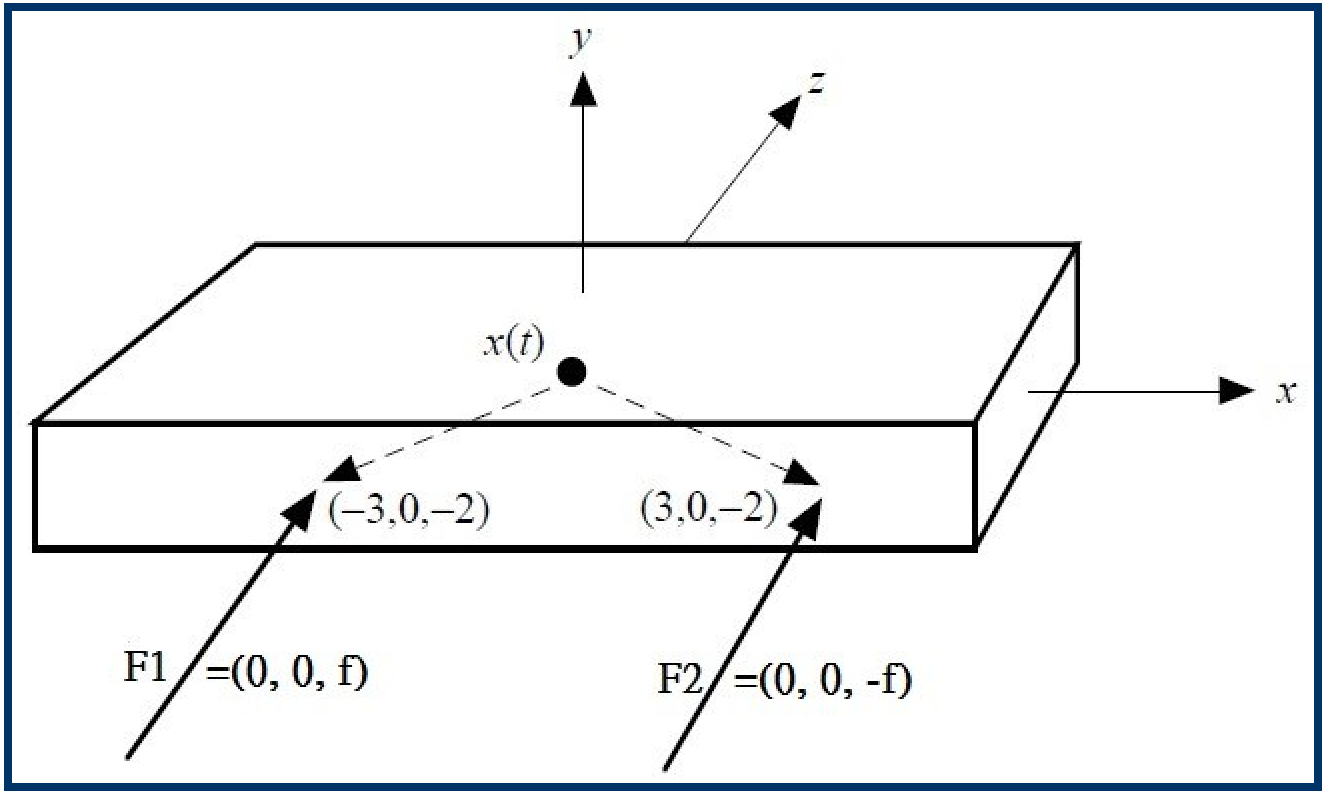
$$= \left(\begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} \right) \times \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} = \mathbf{0}.$$

Examples-translation free movement of a body

- 2 opposite forces acted on two opposite points of the mass center of a body would cause the body to accelerate angularly, without accelerating linearly

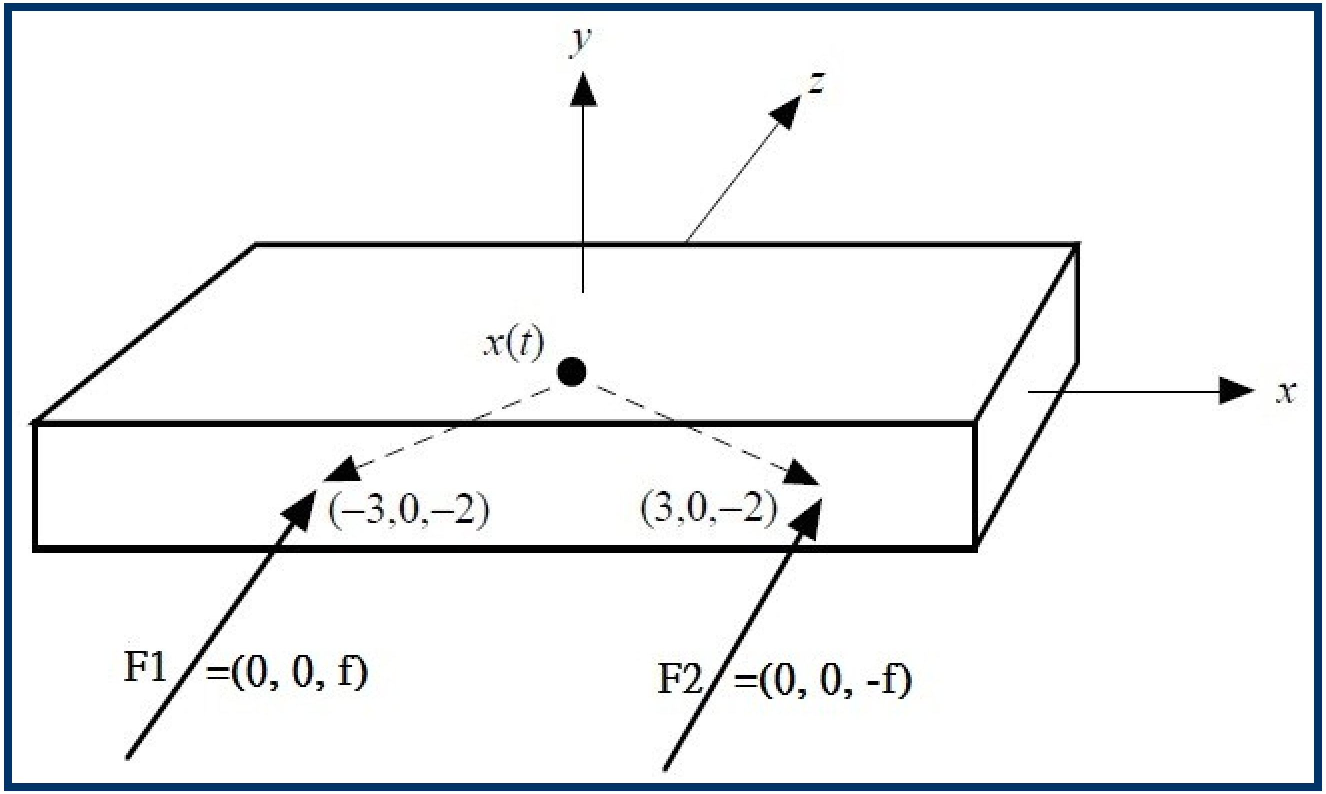
Consider the case shown below:



net force: ***F1+F2=(0,0,0)***

acceleration of mass center: ***0/M =0***

Examples-translation free movement of a body



net **torque:**

$$((x(t)+\begin{pmatrix}-3\\0\\-2\end{pmatrix})-x(t))\times F_1+\ ((x(t)+\begin{pmatrix}3\\0\\2\end{pmatrix})-x(t))\times F_2$$

Examples - rotation free movement of a body

net **torque:**

$$((x(t)+\begin{pmatrix}-3\\0\\-2\end{pmatrix})-x(t))\times F_1+((x(t)+\begin{pmatrix}3\\0\\2\end{pmatrix})-x(t))\times F_2$$

$$=\begin{pmatrix}-3\\0\\-2\end{pmatrix}\times\begin{pmatrix}0\\0\\f\end{pmatrix}+\begin{pmatrix}3\\0\\-2\end{pmatrix}\times\begin{pmatrix}0\\0\\-f\end{pmatrix}$$

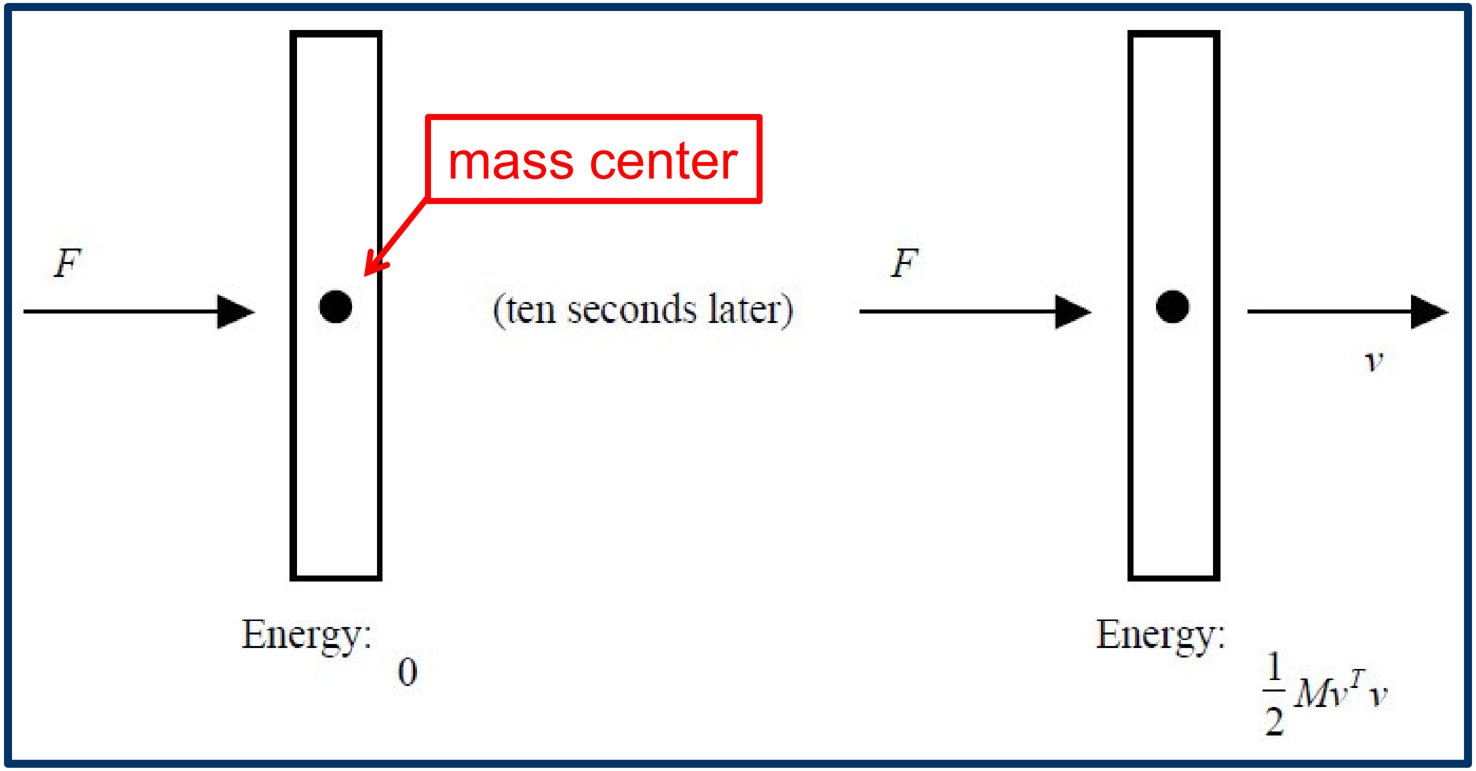
$$=\begin{pmatrix}0\\3f\\0\end{pmatrix}+\begin{pmatrix}0\\3f\\0\end{pmatrix}=\begin{pmatrix}0\\6f\\0\end{pmatrix}.$$

Hence, the forces acting on the block cause it to angularly accelerate about the *y axis.*

Examples - force vs. torque puzzle

- is the following really a paradox?

Case 1:

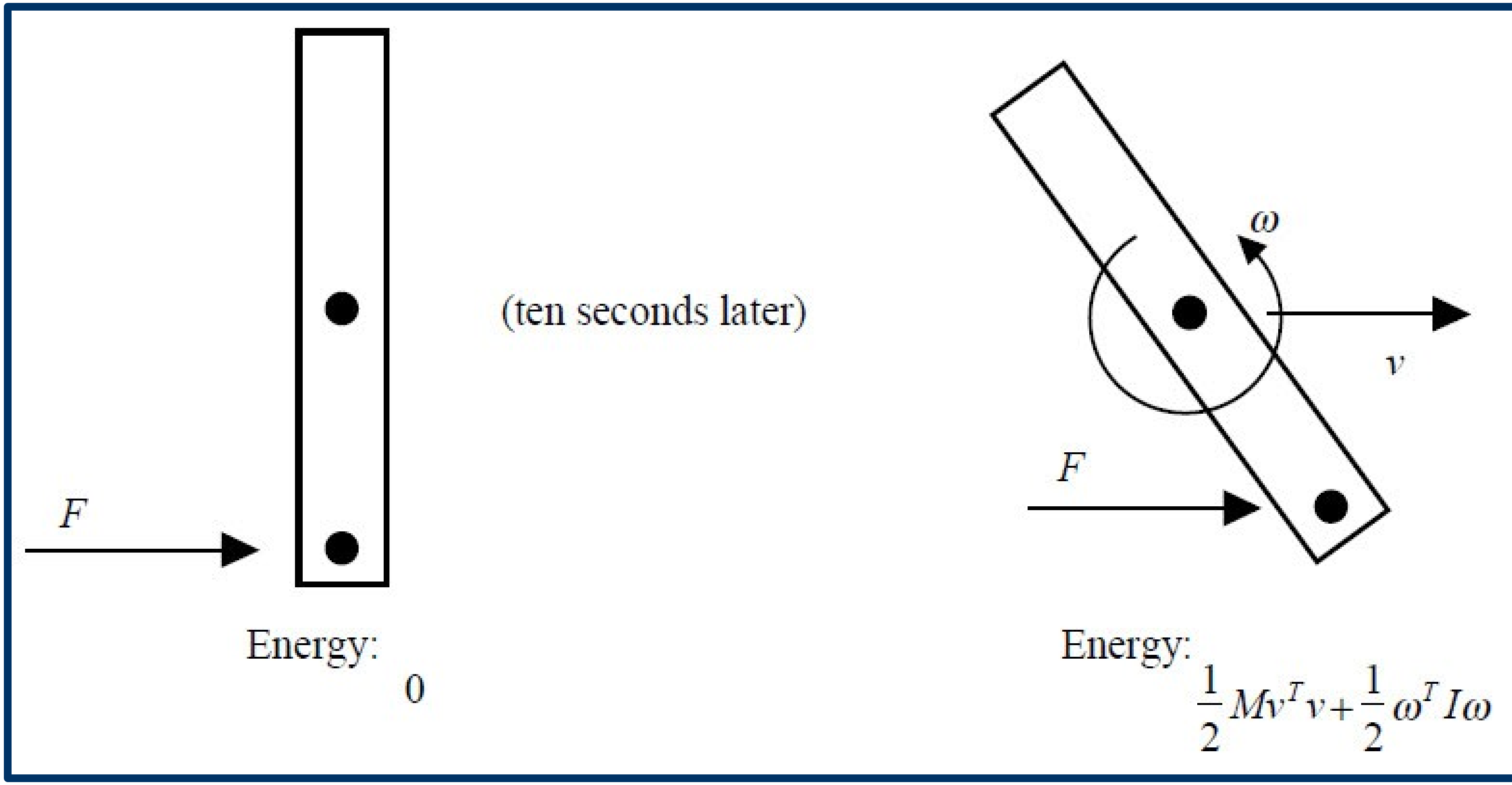


No torque, hence, no angular velocity, but linear velocity *v*. So, kinetic energy = $\frac{1}{2}M\,|\,v\,|^2$

Examples - force vs. torque puzzle

- is the following really a paradox?

Case 2:



with a torque, hence, an angular velocity ω and also a linear velocity *v*. So, kinetic energy = $\frac{1}{2}M|v|^2+\frac{1}{2}\omega^T I\omega$

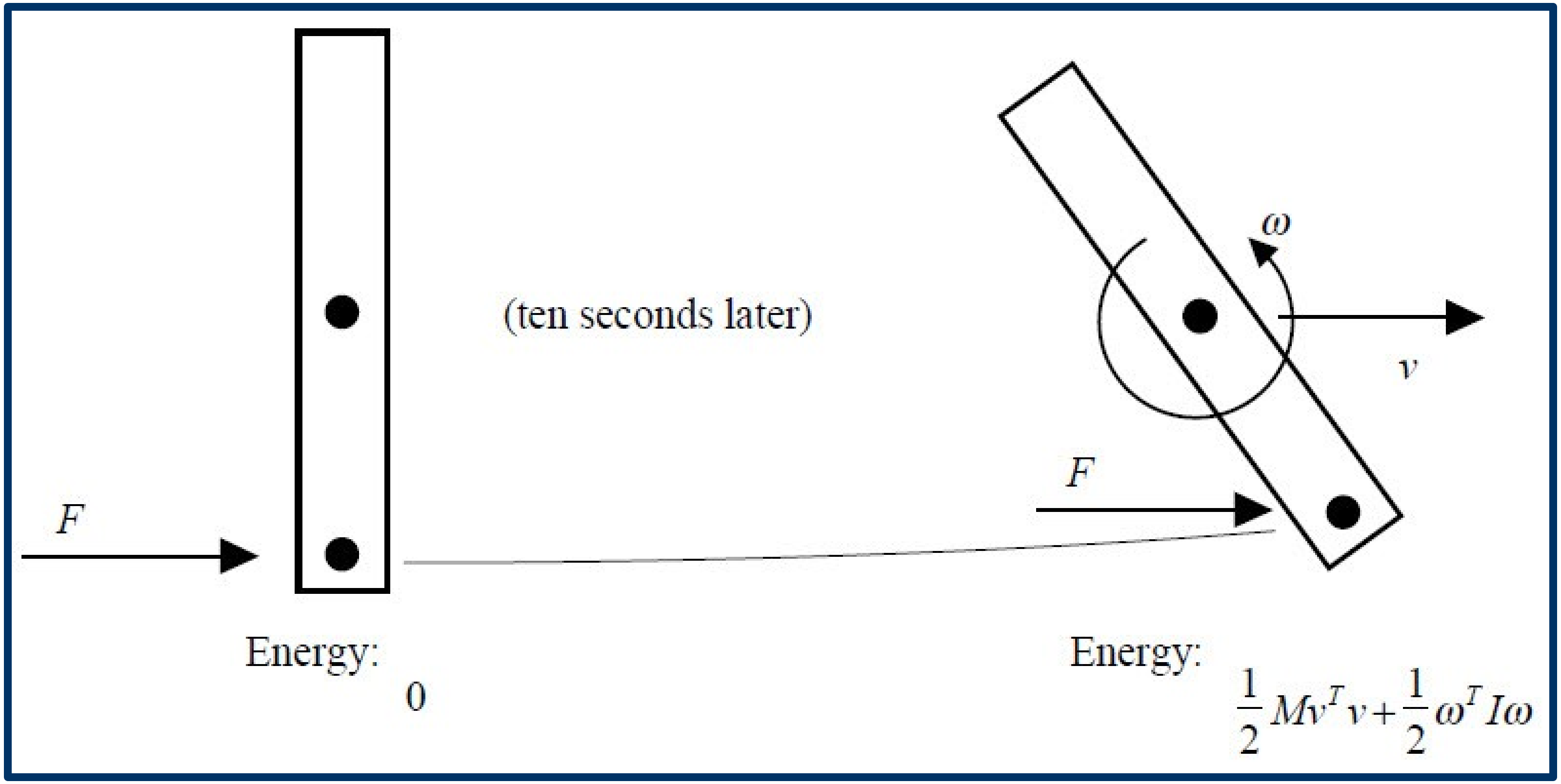
Examples - force vs. torque puzzle

The kinetic energy in the second case is higher than the kinetic energy of the first case.

How can this be possible if identical forces pushed the block in both cases?

Here is why: The kinetic energy of the block is equivalent to the work done by the force. The work done by the force is the integral of the force over the ***path*** traveled in applying that force.

Examples - force vs. torque puzzle



In the 2nd case, the path traced out by the point where the force is applied is clearly longer than the path taken by the center of mass in the 1st case. Thus, when the force is applied off center, more work is done.

End of Physically Based Animation IV