5. Kinematic Linkages

- Concerned with algorithms that use **structured model approach** to produce motion
- motion is controlled by the **model,** not the **animator**
- consider **kinematic models** and **dynamic models**
- **kinematic control:** ← Properties of motion
  movement of objects irrelevant to the forces in producing the movement
- **dynamic control:** ← Rules governing the interactions
  concerned with computing the underlying forces that are used to produce movement

Kinematics

- describes the motion of points, bodies (objects) and systems of bodies (groups of objects) without consideration of the causes of motion
- kinematics studies the trajectories of points, lines and other geometric objects and their differential properties such as velocity and acceleration.
- can be abstracted into purely mathematical functions. E.g., rotation can be represented by elements of the unit circle in the complex plane.

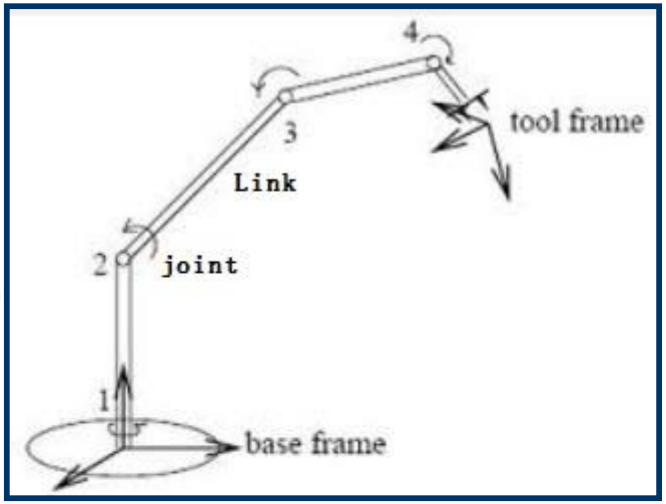
Dynamics

- study how a physical system might develop or alter over time and study the causes of those changes.
- concerned with the study of forces and torques and their effect on motion
- Isaac Newton defined the fundamental physical laws which govern dynamics in physics, especially his second law of motion

Hierarchical Kinematic Modeling

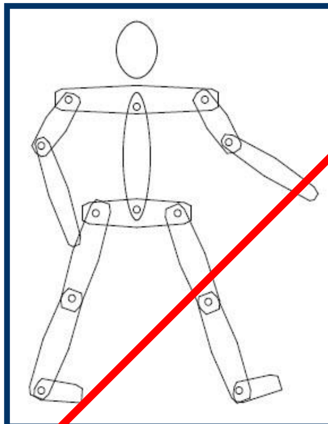
- enforcement of connectivity constraints among objects organized in a tree-like structure

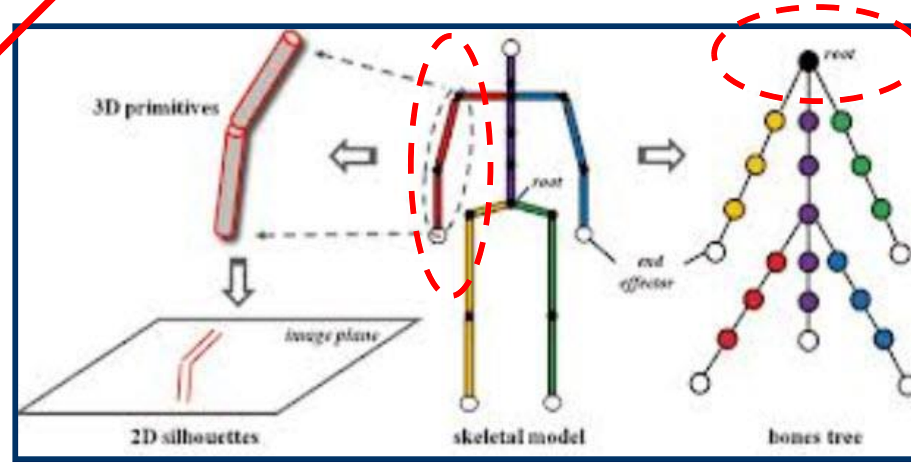
e.g. robotic manipulator



two or more sections connected by a flexible joint

e.g. multibody joint chain





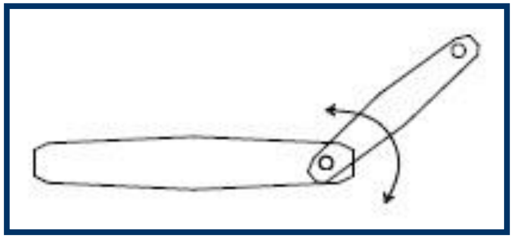
Articulated figure/structure

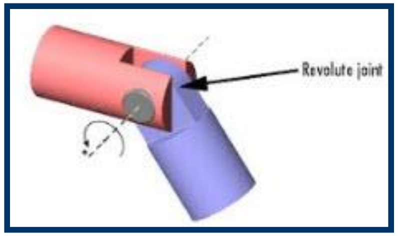
Articulation: movement of an **appendage** by changing the configuration of a joint

A figure is generated by manupulating the **objects of the limbs**

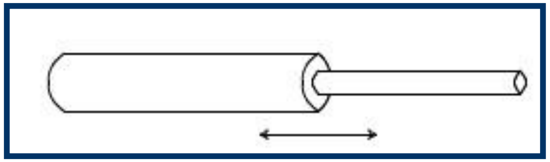
Joint Types:

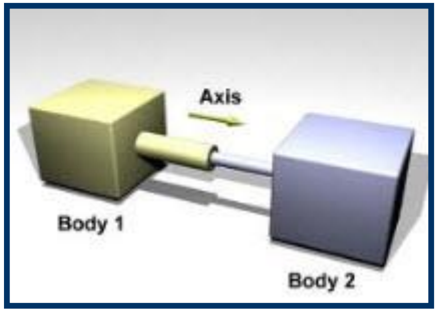
- Revolute joint



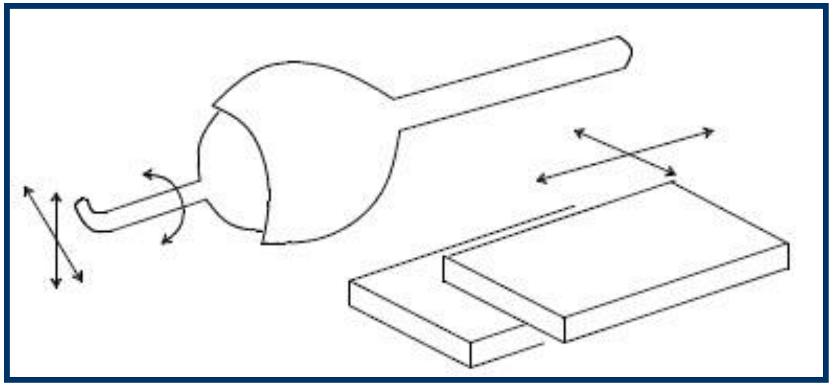


- Prismatic joint





- Complex joints (ball-and-socket joint, planar joint)

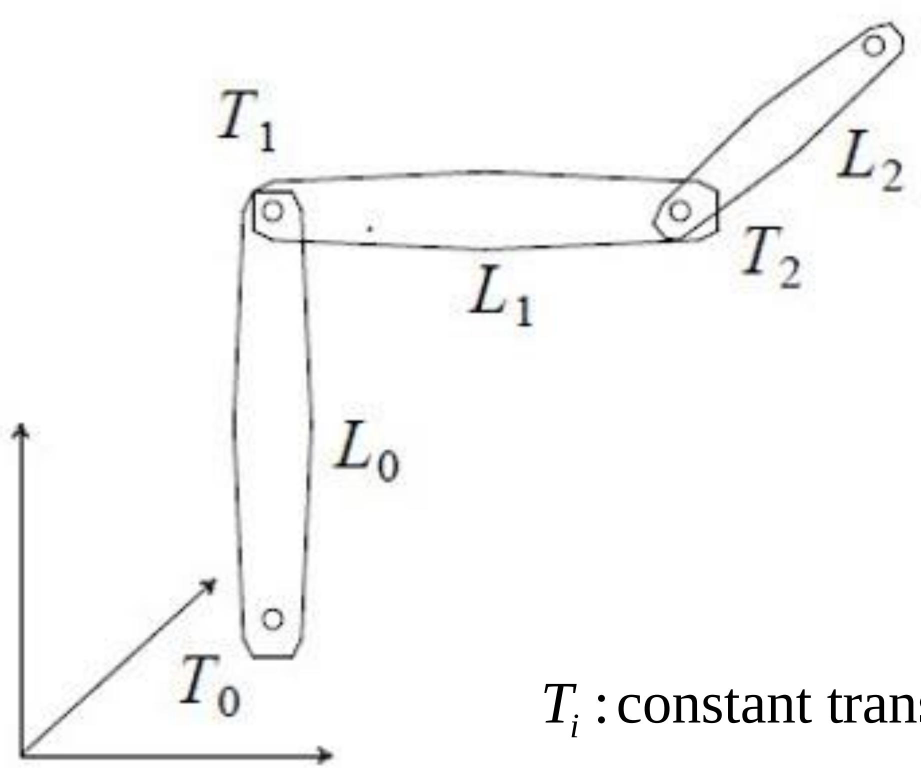


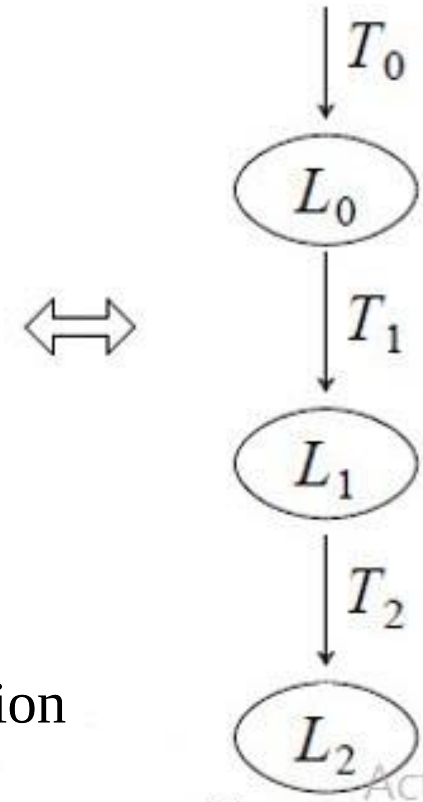
Data Structure for Hierarchical Modeling:

Hierarchical Structure ↔ *Tree Structure*

object part (link) ↔ *node*

joint ↔ *arc*

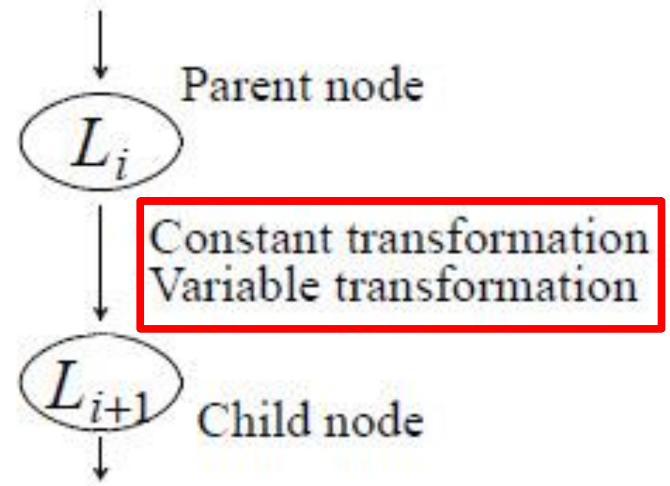




T_i : constant transformation

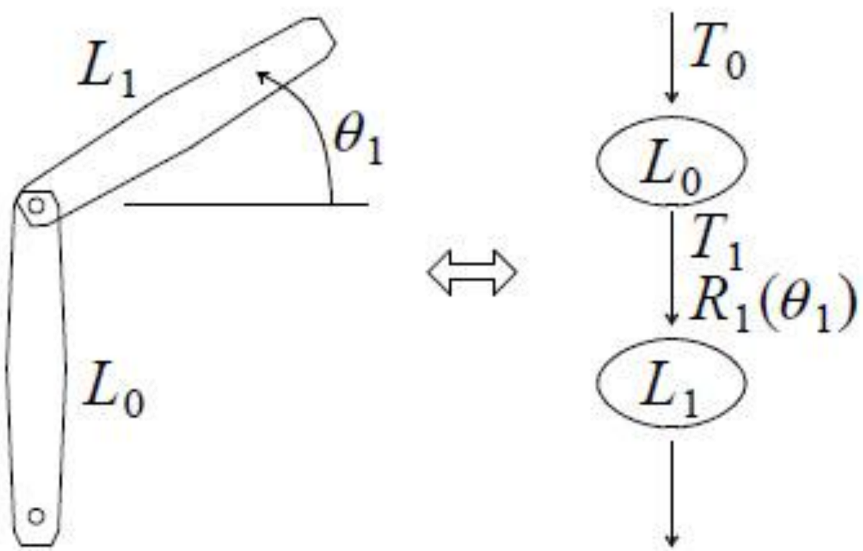
Constant transformation (translation + rotation):

-transform *link(i + 1) to its neutral position* relative to *link(i)*



Variable transformation (rotation):

- responsible for actual joint articulation



Forward Kinematics

- given the joint angles, determine the position and orientation of the end effector
- traverse the corresponding tree following a depth-first pattern
- use a *stack to concatenate and restore* transformations
- given 2 key positions, interpolating joint angles of the key positions to generate intermediate end effector positions (and orientations) might not work well
- a better approach is to interpolate end effector's positions and orientations of the key positions (and compute the corresponding joint angles)

Local Coordinate Frames

Local coordinate frame: local coordinate system associated with a joint

Need well-defined method for converting coordinates of a point from one frame to another (especially the global coordinate system) for

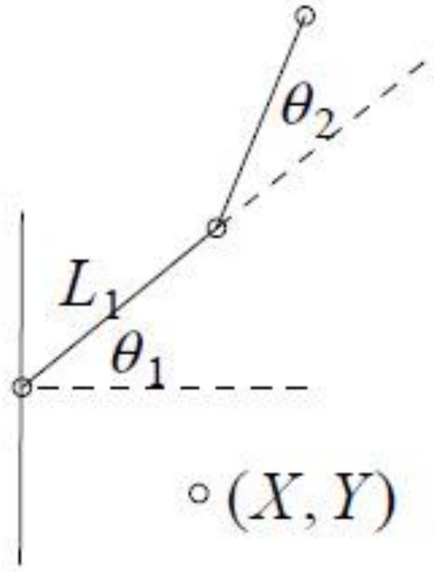
display purposes !

(so the data structure is especially important here)

Inverse Kinematics

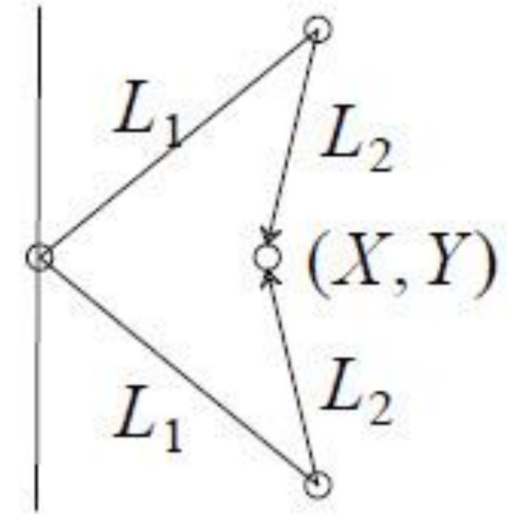
- Given the desired position and orientation of the *end effector,* find the joint angles required to attain that configuration (can have zero, one or more solutions)

Overconstrained system*:* too many constraints on the configuration that no solution exists

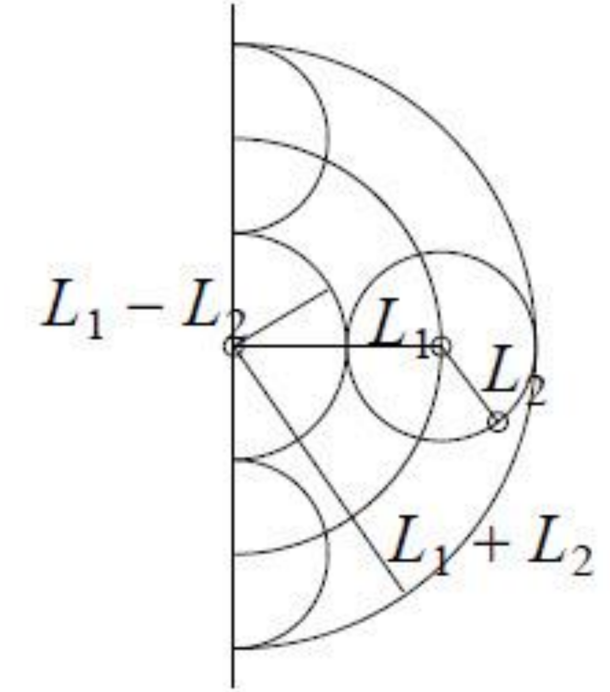


$0 \leq \theta_i \leq 45^o$

Underconstrained system: relatively few constraints on the configuration and there are many solutions to the problem posed



Reachable workspace: volume the end effector can reach



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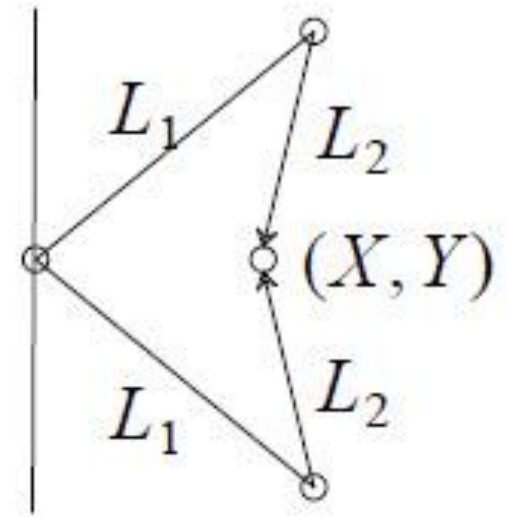

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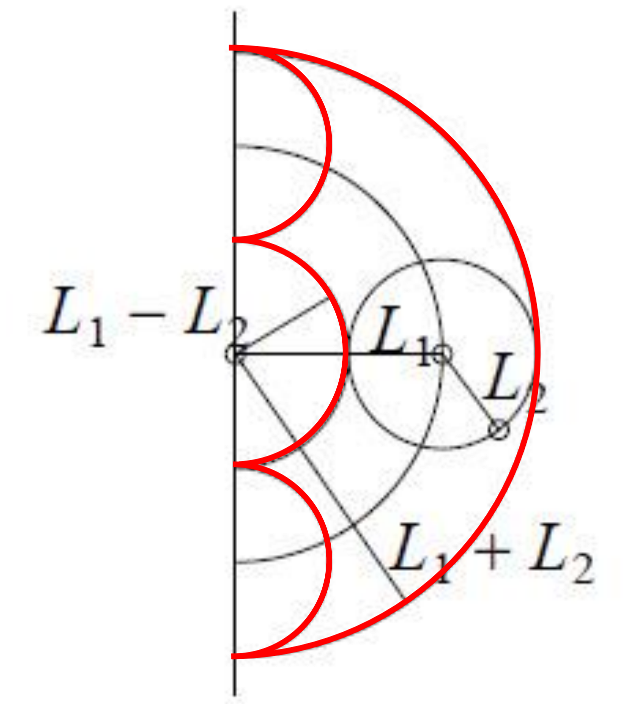
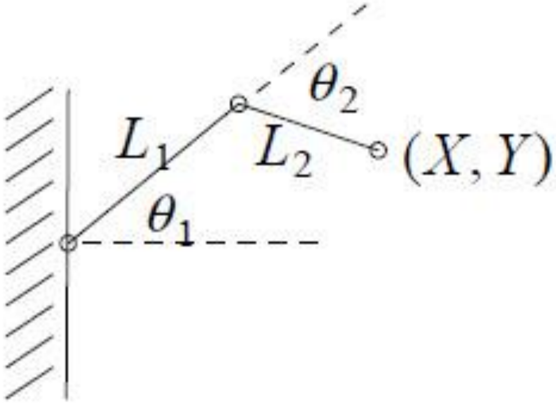


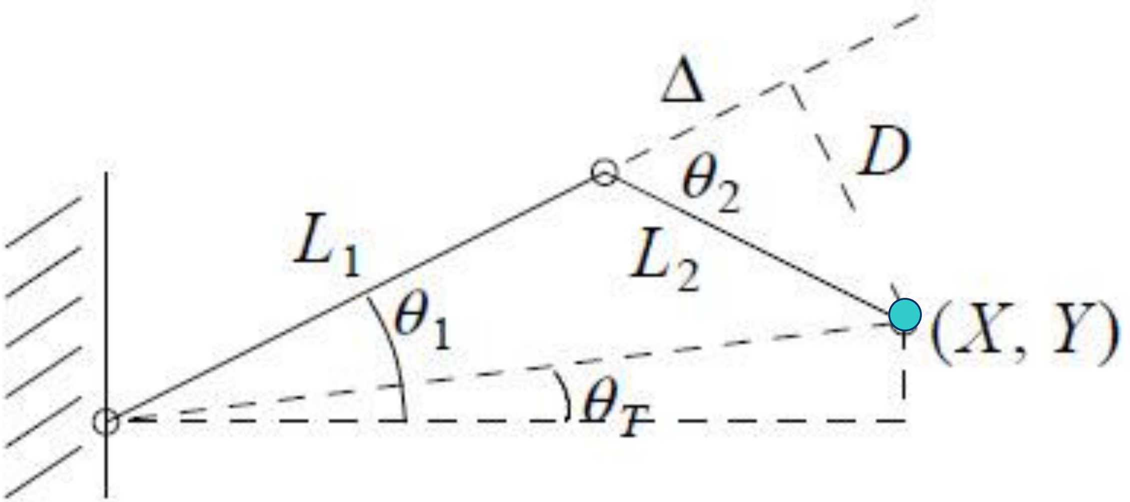
Solving a simple system by analysis

Consider the 2-link arm in 2D space:



Given the desired position (X, Y) for the end effector $(L_1 - L_2 \leq \sqrt{X^2 + Y^2} \leq L_1 + L_2)$ find joint angles θ_1 and θ_2 so that the corresponding position of the end effector is (X, Y).

Analysis:



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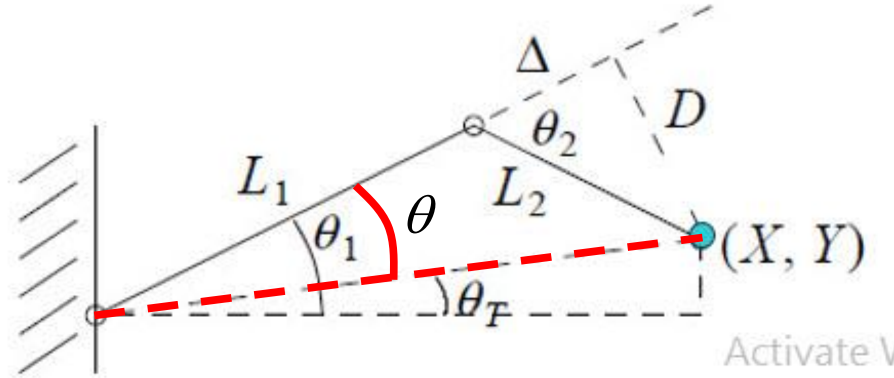


Using the fact that

$$\Delta = \frac{X^2 + Y^2 - L_1^2 - L_2^2}{2L_1}$$

$$X^2 + Y^2 = (L_1 + \Delta)^2 + D^2$$

$$L_2^2 = \Delta^2 + D^2$$

and

$$\theta_T = \cos^{-1}\left(\frac{X}{\sqrt{X^2 + Y^2}}\right)$$

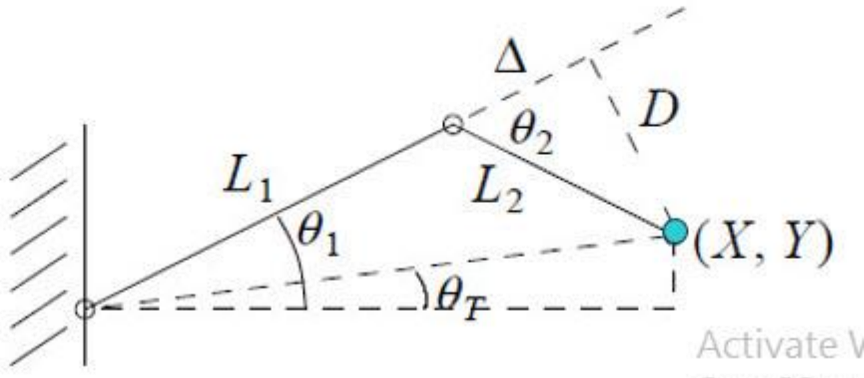
$$\cos\theta = \frac{L_1 + \Delta}{\sqrt{X^2 + Y^2}}$$

$$= \frac{X^2 + Y^2 + L_1^2 - L_2^2}{2L_1\sqrt{X^2 + Y^2}}$$

we have

$$\theta_1 = \cos^{-1}\left(\frac{L_1^2 + X^2 + Y^2 - L_2^2}{2L_1\sqrt{X^2 + Y^2}}\right) + \theta_T$$

Solving a simple system by analysis:



On the other hand, we have

$$\cos\theta_2 = \frac{\Delta}{L_2} = \frac{X^2 + Y^2 - L_1^2 - L_2^2}{2L_1L_2}$$

Hence,

$$\theta_2 = -\cos^{-1}\left(\frac{X^2 + Y^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$$

However, analytic solutions are not tractable in many cases.

End of Kinematic I