4. Interpolation-Based Animation

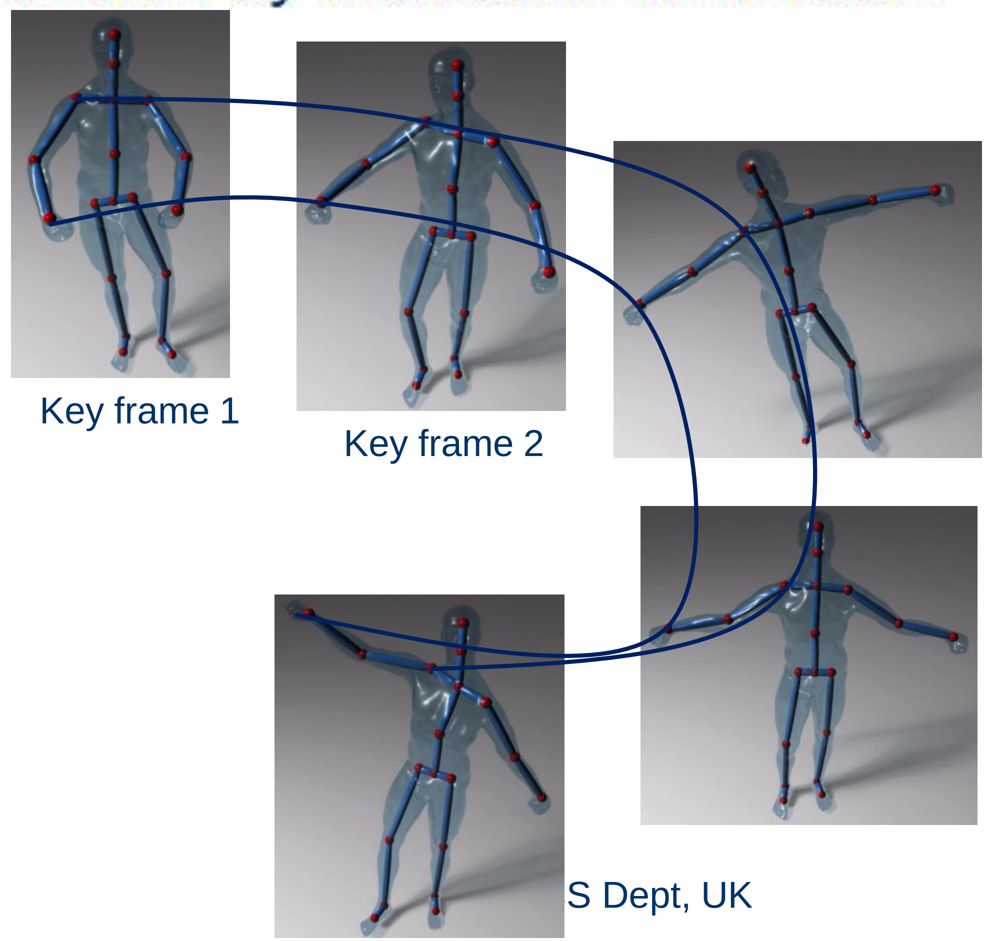
Methods for precisely specifying the motion of objects using basics introduced in previous chapter :

- Key-frame animation systems
- Animation languages
- Object deformation

Key-Frame Systems (good reference: Maya)

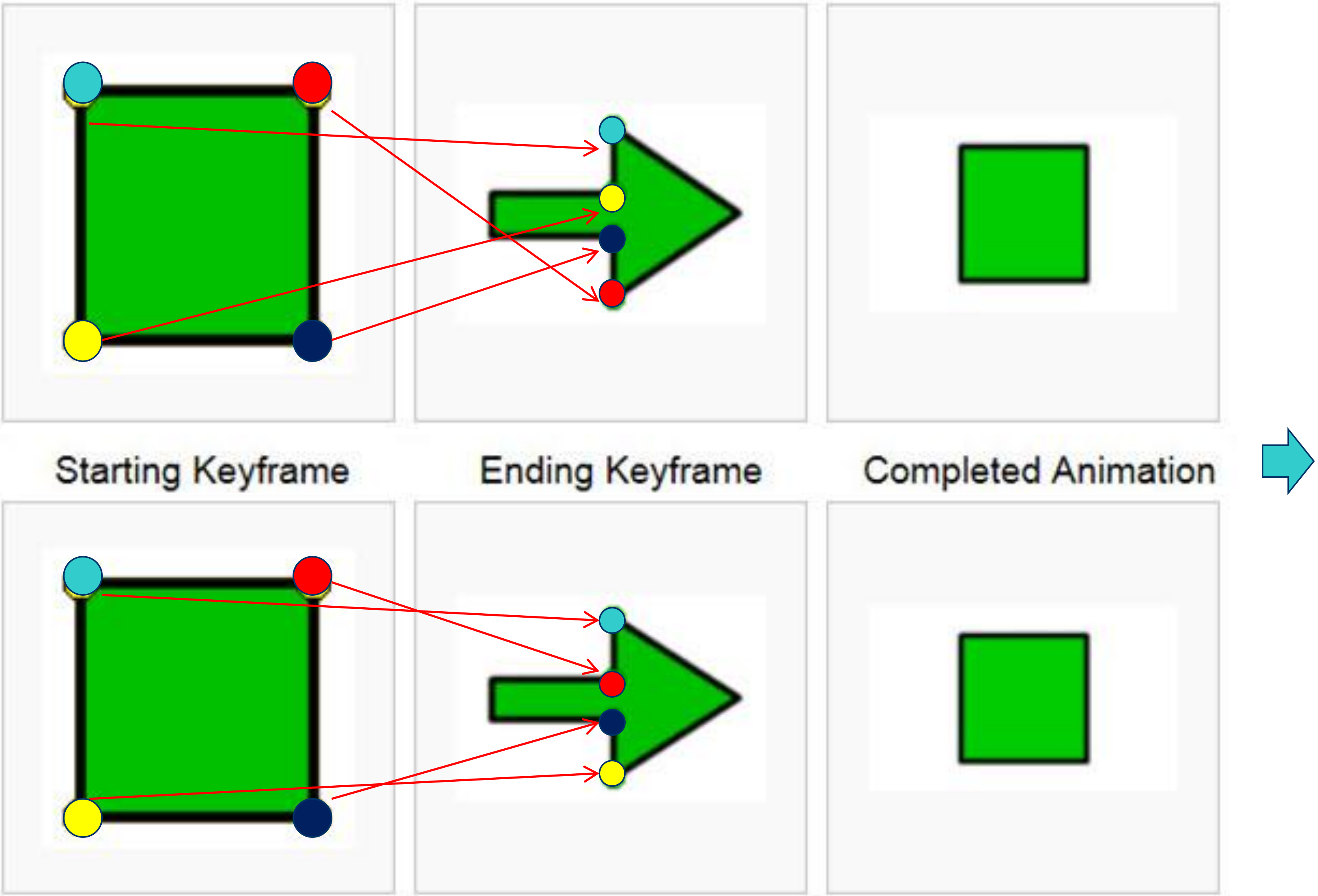
In a typical animation system, the *key frames* are defined & drawn by the master animators, interme-diate frames are then drawn by assistant animator.

Relatively simple in computer animation if the shapes are defined by polygons and the correspond-ence between the vertices is known.



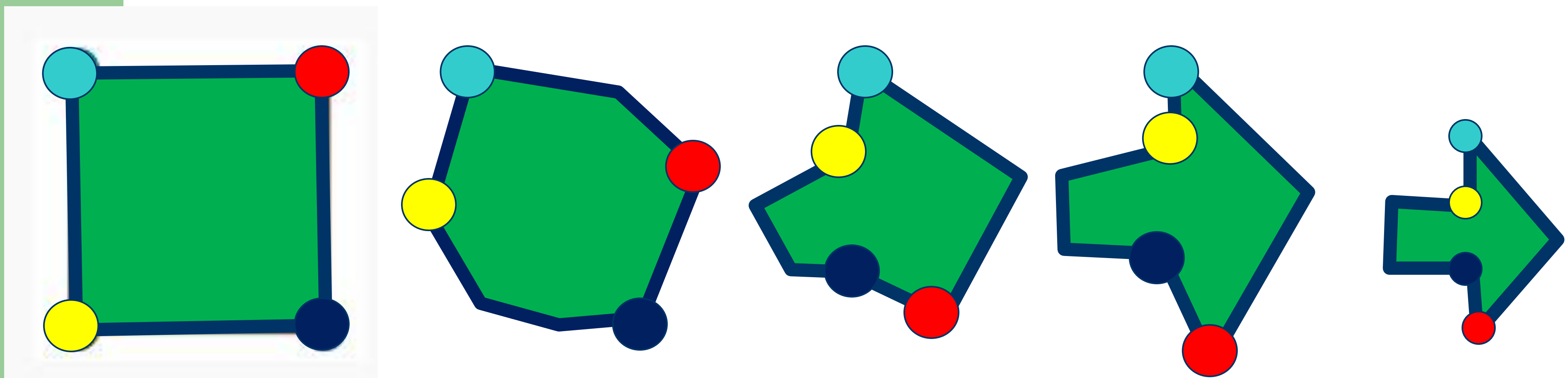
Key-Frame Systems

Order of correspondence is important

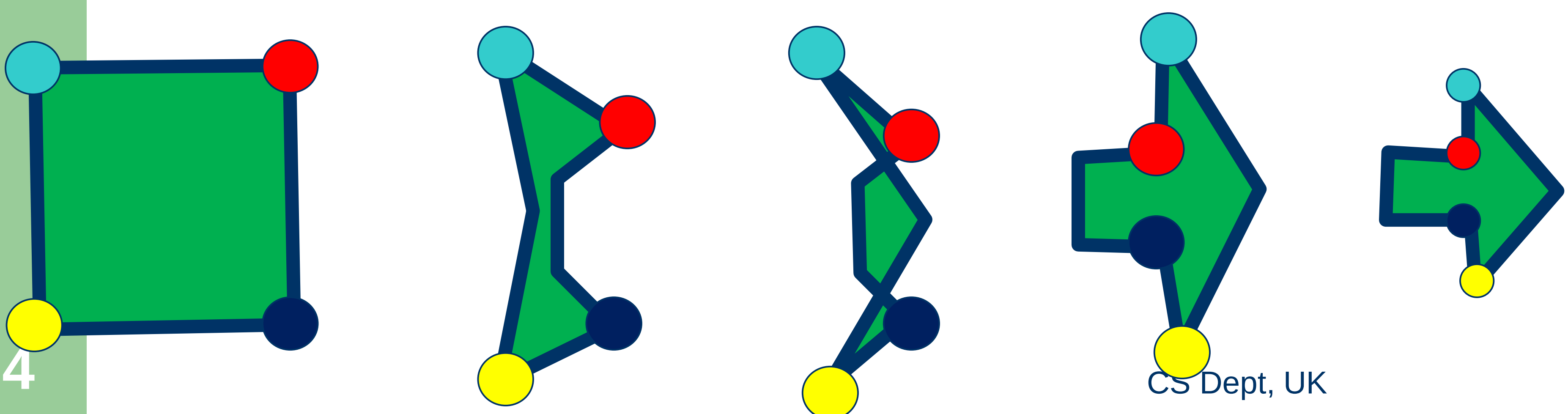


Key-Frame Systems

Order of correspondence is important



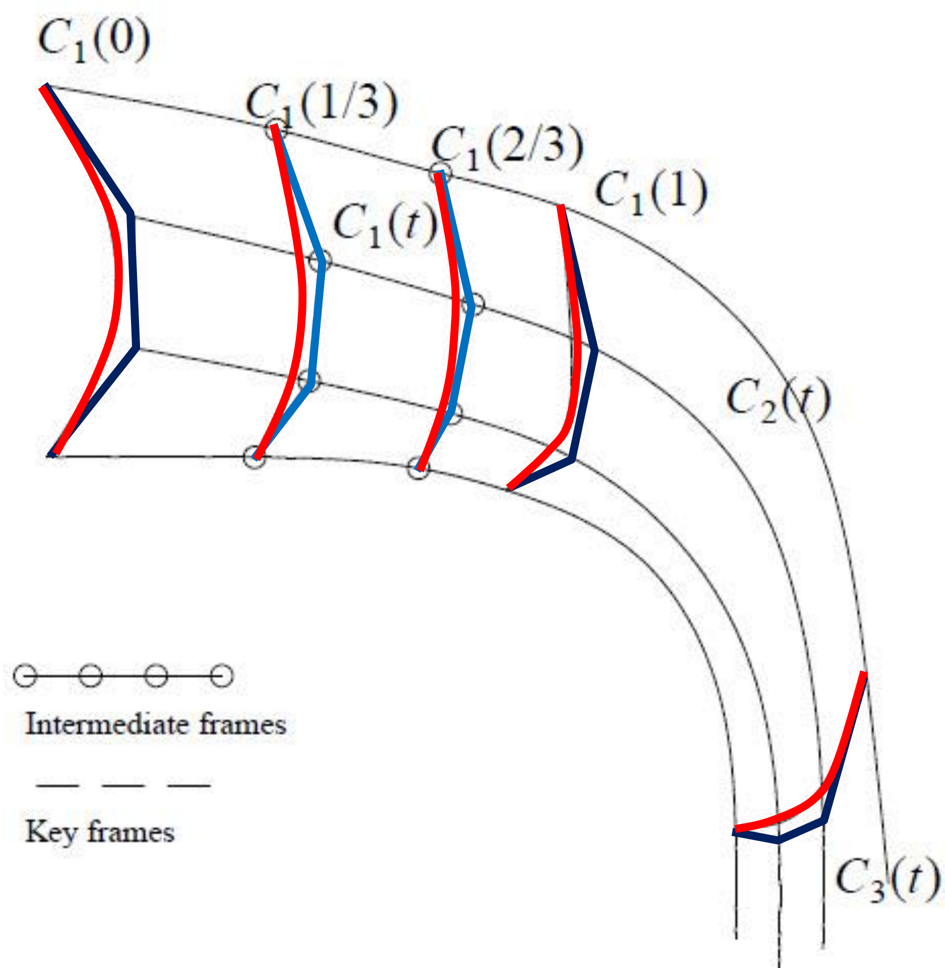
Starting frame **ending frame**



Key-Frame Systems

For shapes defined by piecewise curves such as *composite Bezier curves* or *B-spline curves*, intermediate frames can be generated using *surface patch* technique:

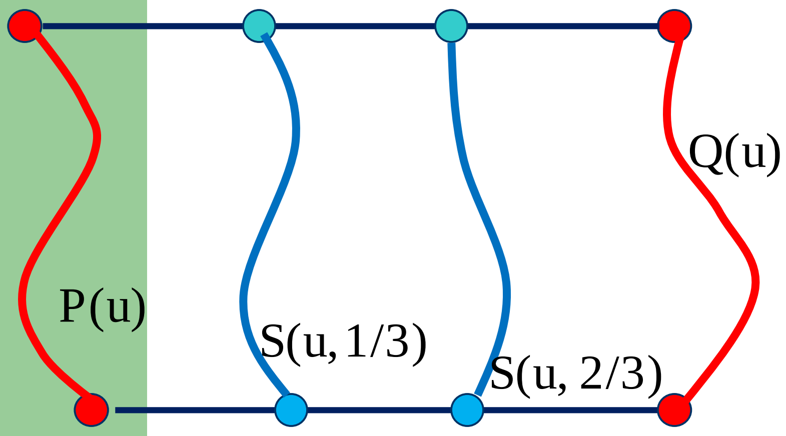
(i) Interpolating control points of curves in the key frames



Key-Frame Systems

(ii) The path between two adjacent key frames is already known

(a) the path is a straight line



Construct a surface patch as follows:

$$\mathbf{S}(u,v) = \mathbf{P}(u)(1 - b(v)) + b(v)\,\mathbf{Q}(u)$$

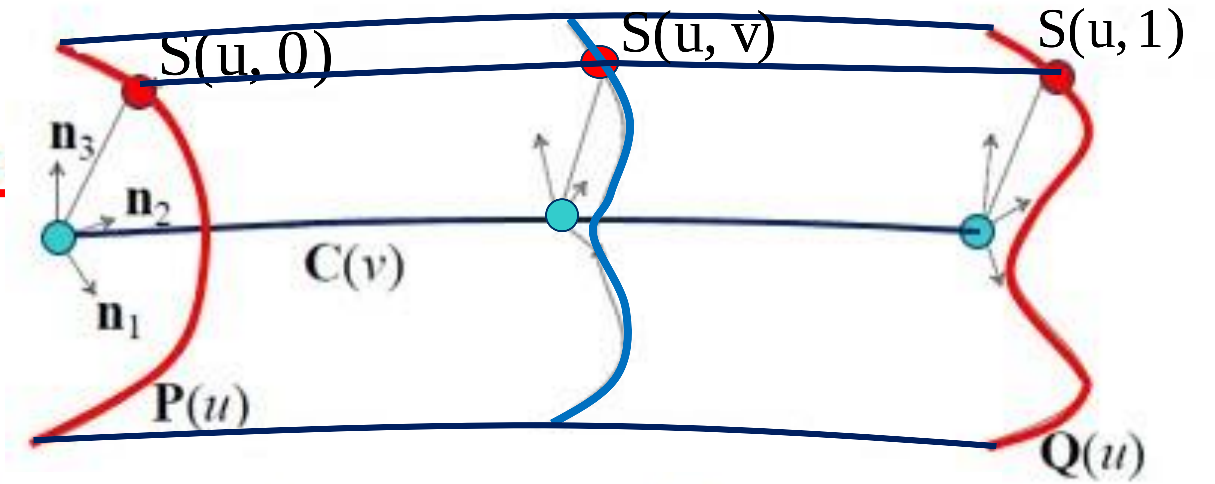
$$0 \le u,\, v \le 1$$

where

$$0 \le b(v) \le 1 \qquad b(0) = 0, \qquad b(1) = 1$$

$$\left(e.g., \quad \begin{aligned} b(v) &= v \\ b(v) &= 3v^2(1-v) + v^3 \end{aligned} \right)$$

(b) The path is a cubic curve segment (Bezier, B-spline)



$$
\begin{aligned}
\mathbf{S}(u,v) = \mathbf{C}(v) &+ \{(1-\alpha(v)[\mathbf{n}_1(0)\ \textit{component of}\ \mathbf{P}(u)-\mathbf{C}(0)] \\
&+ \alpha(v)[\mathbf{n}_1(1)\ \textit{component of}\ \mathbf{Q}(u)-\mathbf{C}(1)]\}\mathbf{n}_1(v) \\
&+ \{(1-\alpha(v)[\mathbf{n}_2(0)\ \textit{component of}\ \mathbf{P}(u)-\mathbf{C}(0)] \\
&+ \alpha(v)[\mathbf{n}_2(1)\ \textit{component of}\ \mathbf{Q}(u)-\mathbf{C}(1)]\}\mathbf{n}_2(v) \\
&+ \cdots
\end{aligned}
$$

where $\alpha(v)$ is a blending function and $\mathbf{n}_1(v)$, $\mathbf{n}_2(v)$ and $\mathbf{n}_3(v)$ are Frenet frame axes at $\mathbf{C}(v)$

4.2 Animation Languages

What are animation languages?

- *Structured commands* used to encode the information necessary to produce animations

Why do we need animation languages?

- To avoid *overhead* in producing motion sequences

4.2 Animation Languages

Script-based or *graphical*

- *Script-based*: composed of text instructions, earlier approaches such as ANIMA II, AL,

```
set position<name><x><y><z>at frame<number>
set rotation<name>[x,y,z]to<angle>at frame<number>
change position<name>to<x><y><z>from frame<number>to  frame<number>
change rotation<name>[x,y,z]by<angle>from frame<number>to frame<number>
```

4.2 Animation Languages

ANIMA II: released by Reverse System in 2005

Anima II 東方


AL: developed by Steve May at the Advanced Computing Center for the Arts and Design (ACCAD) which is a part of The Ohio State University. Steve is currently employed at Pixar Animation Studios and maintains AL in his spare time.

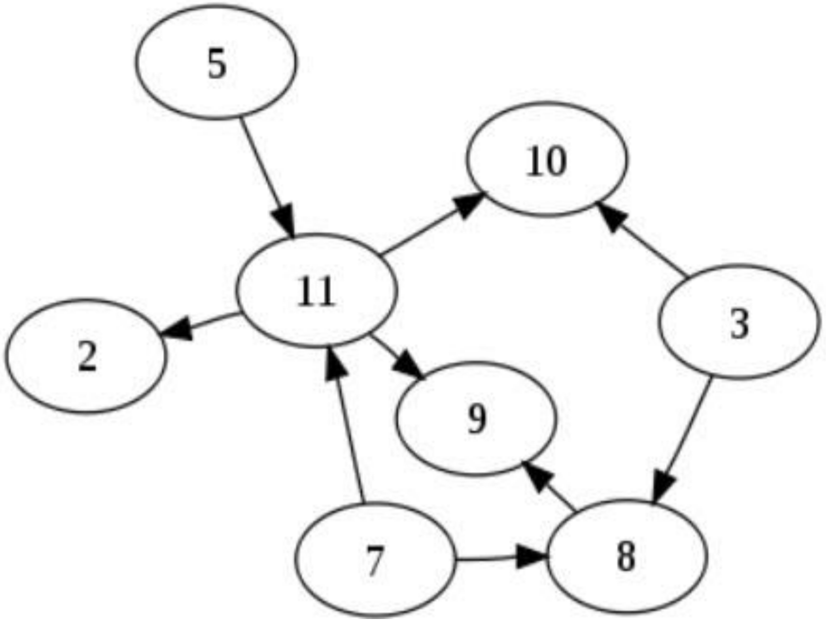
Download site: http://accad.osu.edu/~smay/AL/download.html

4.2 Animation Languages

- *graphical* : encoding relationships between objects and procedures using acyclic graphs

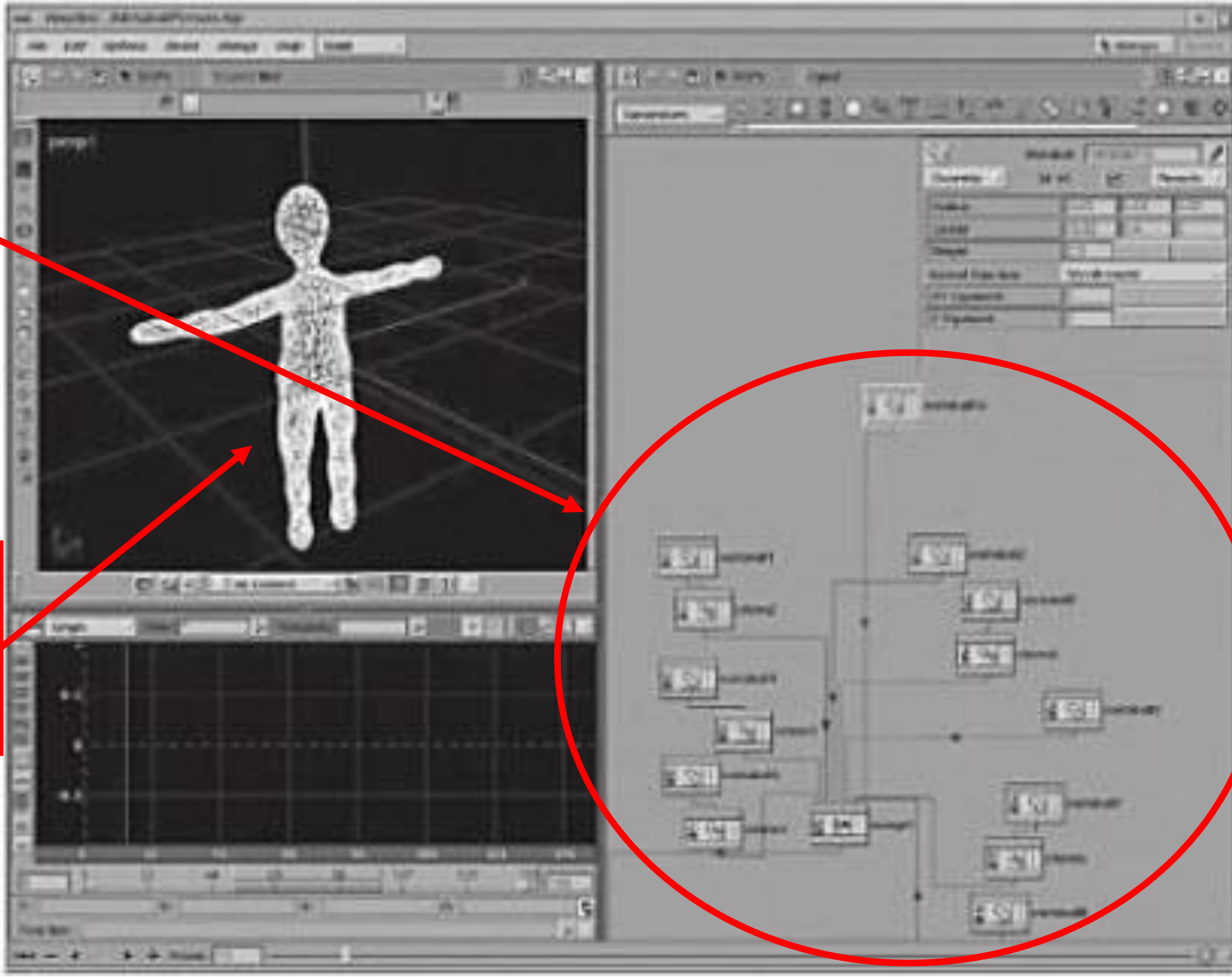
An animation is represented by a dataflow network

such as *Houdini, Maya* and most of recent animation languages



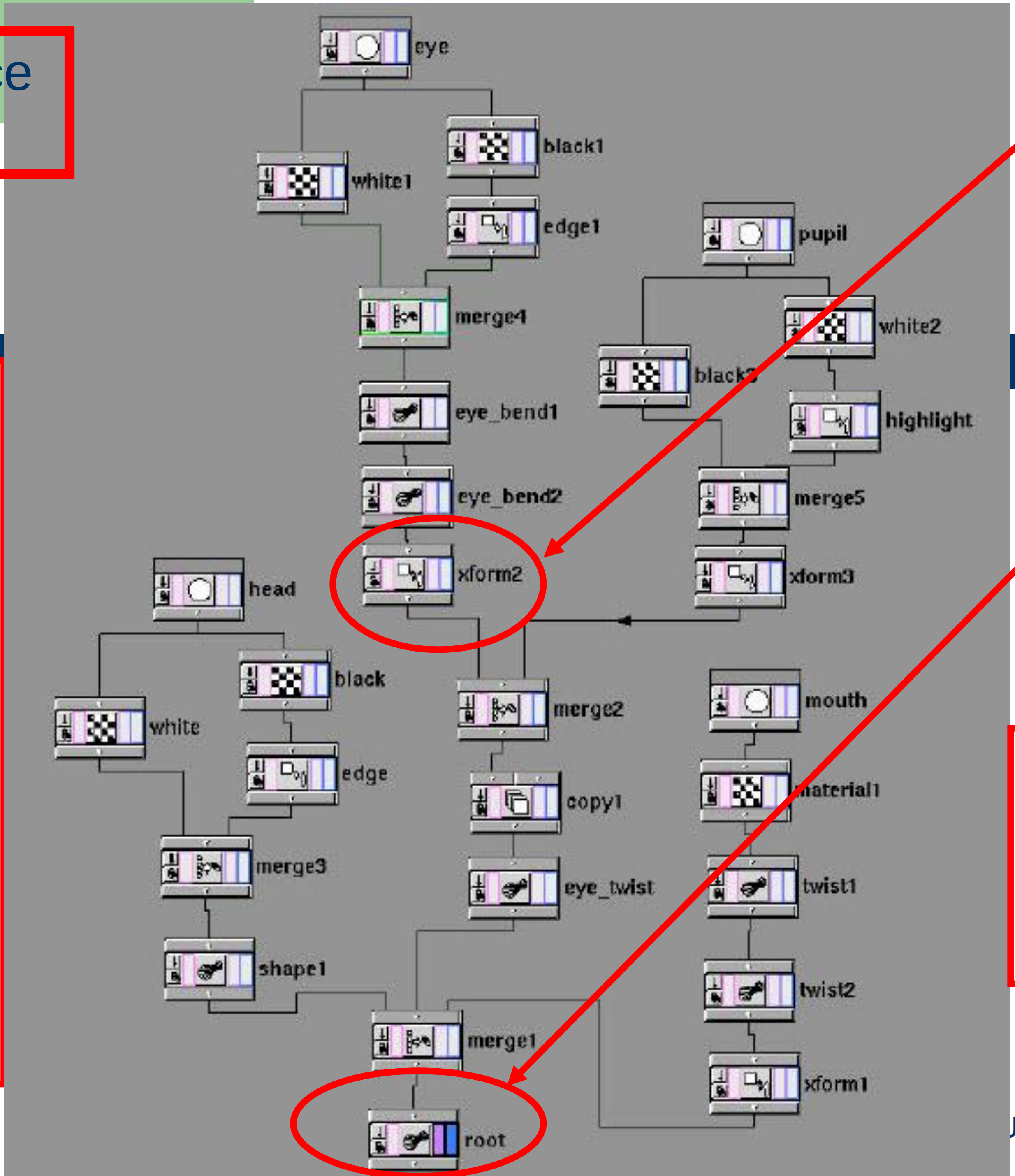
Houdini dataflow network

The object it generates



Cartoon face prototype

collection of nodes connected together that describe the steps needed to accomplish a task



Operator node

Container node

Hierarchy of nodes is created using container nodes

Advantages of Animation Languages

- Bigger *reusebility*, *mobility*, and *alterability*: The script can be used at any time to regenerate it, can be copied and transmitted easily, allows the animation to be iteratively refined (because the script can be incremently changed and a new animation generated)

- The availability of programming constructs allows an algorithmic approach to motion control

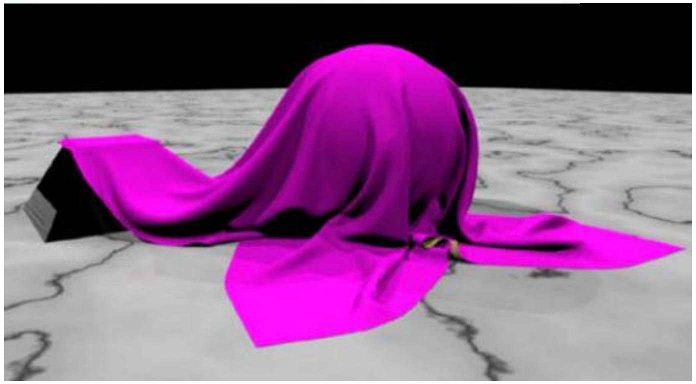
You don't have much choice, but to follow the physics

4.3 Deforming Objects

- Physically based deformation
  - simulating the flexing of objects undergoing forces

Bending

Dripping



Cloth modeling: mass-spring system

4.3 Deforming Objects

- Physically based deformation

1. Subdividing surface S(u,v)
2. Scaling S(u,v) with T_s
3. Relocating components that do not change shape $S \circ C_i(t)$
4. Set up feature preserving objective function
5. Set up constraints
6. Performe constrained shape deformation
7. Rendering

4.3 Deforming Objects

- Physically based deformation

The deformation process requires the construction of a feature-preserving objective function.

This function is used to determine the shape of the deformed object in an optimization process. Hence, the objective function should be defined as the difference of these two surfaces.

$$V(u,v) = (\bar{S} - T_s \circ S)(u,v)$$

New surface

External force

4.3 Deforming Objects

- Physically based deformation

$$V(u,v) = (\bar{S} - T_s \circ S)(u,v)$$

The goal is to minimize the energy of the displacement function

$$E(V) = E_{bending} + E_{gravity} + E_{spring} + E_{moment} + E_{edgeforce}$$

$\boxed{\text{Set to zero}}$

4.3 Deforming Objects

- Physically based deformation

$$V(u,v) = (\bar{S} - T_s \circ S)(u,v)$$

$$E_{bending} = \iint_D \kappa \left[\frac{1}{2}(V_{uu} + V_{vv})^2 - (1-\sigma)(V_{uu}V_{vv} - V_{uv}{}^2) \right] dudv$$

$$E_{stretching} = \frac{1}{2} \iint_D \left[(2G+\lambda)(V_u{}^2 + V_v{}^2) + 2\lambda(V_u V_v) \right] dudv$$

$$E_{spring} = \frac{1}{2} \iint_D K(\tau_i + \sigma_i)[V(\tau_i, \sigma_i)]^2 dudv$$

Substitute and minimize

Imaging your image is made of rubber and then warp your rubber

4.3 Deforming Objects

- User defined distortion
  - simulator deforms the object directly and defines key shapes
  - usually needs to use non-affine transformations
    - *warping*
    - *coordinate grid deformation*
  - an affine transformation is a linear mapping from an affine space to an affine space

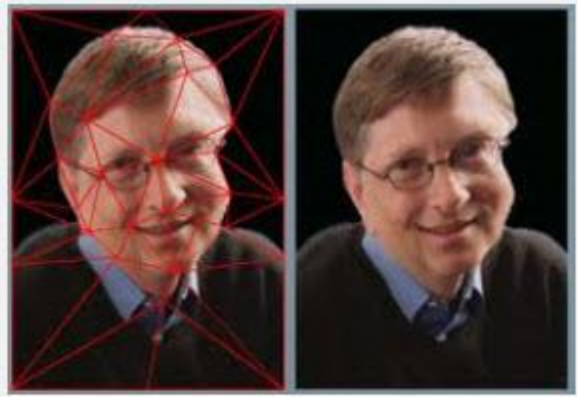
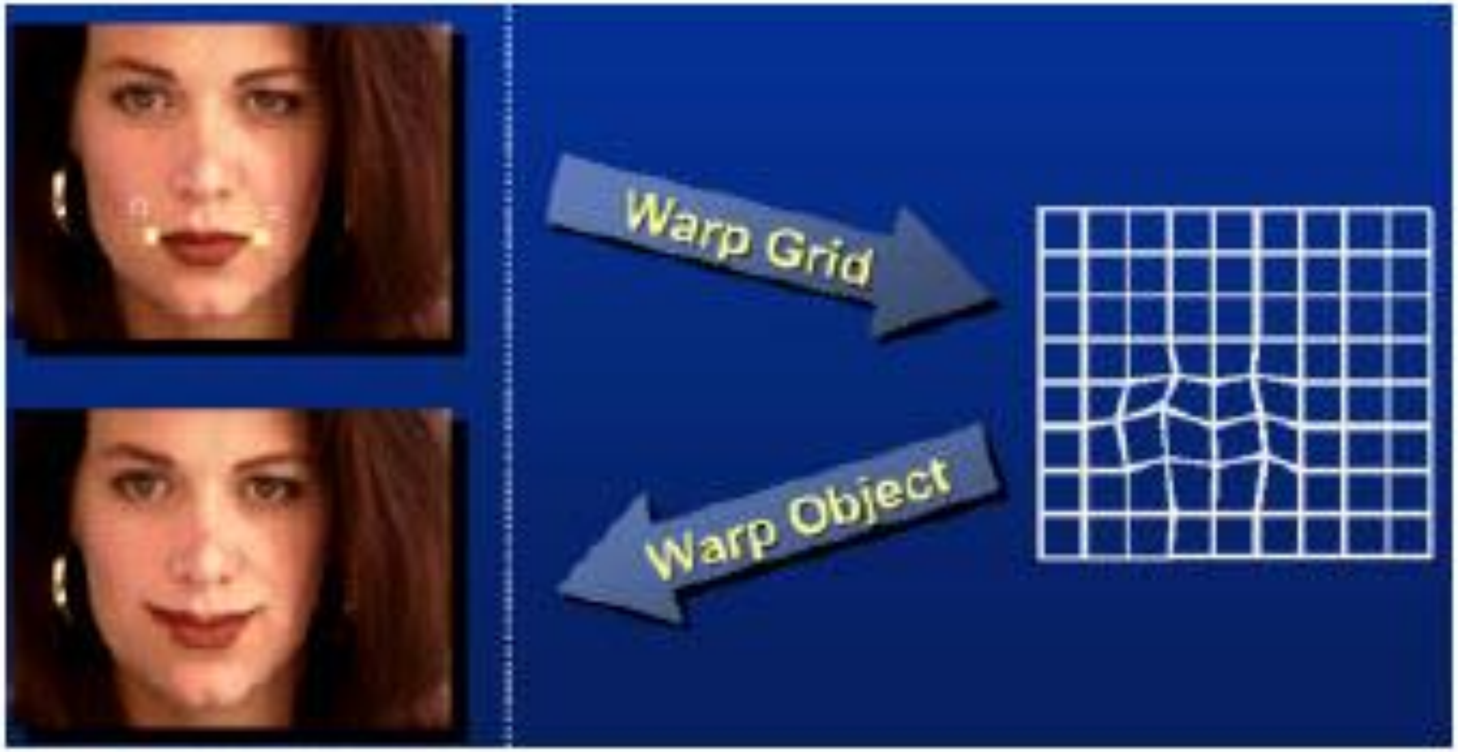


Image Warping – non-parametric

- Move control points to specify a spline warp
- Spline produces a smooth vector field



Warp specification - dense

- **How can we specify the warp?**

Specify corresponding *spline control points*

  - *interpolate* to a complete warping function

But we want to specify only a few points, not a grid

Warp specification - sparse

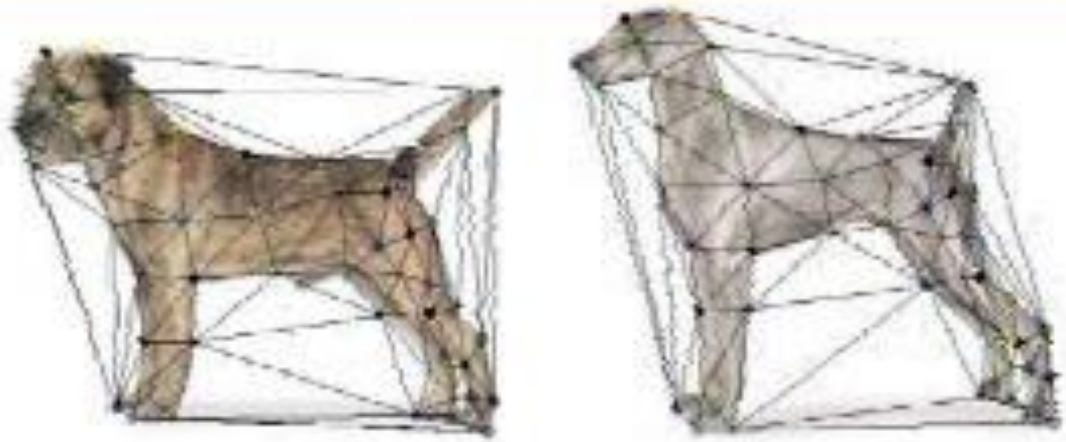
- **How can we specify the warp?**

Specify corresponding *points*

  - *interpolate* to a complete warping function
  - How do we do it?

How do we go from feature points to pixels?

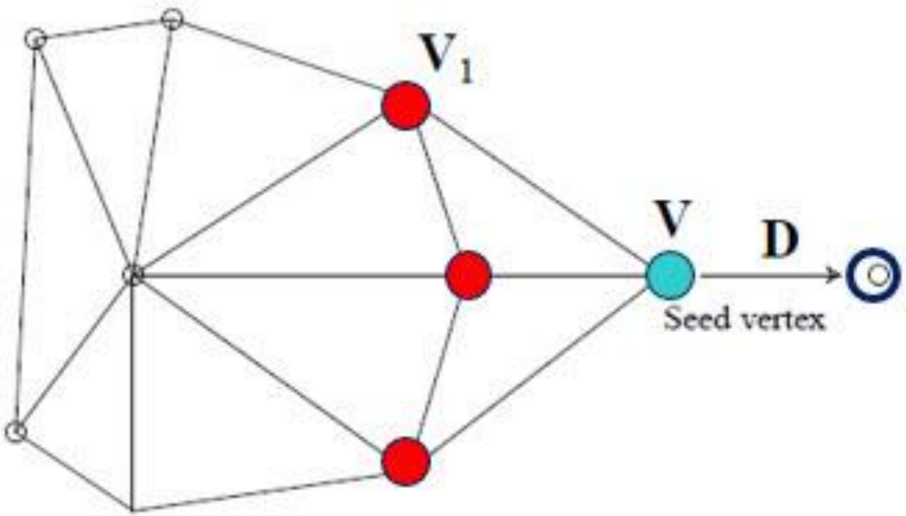
Triangular Mesh



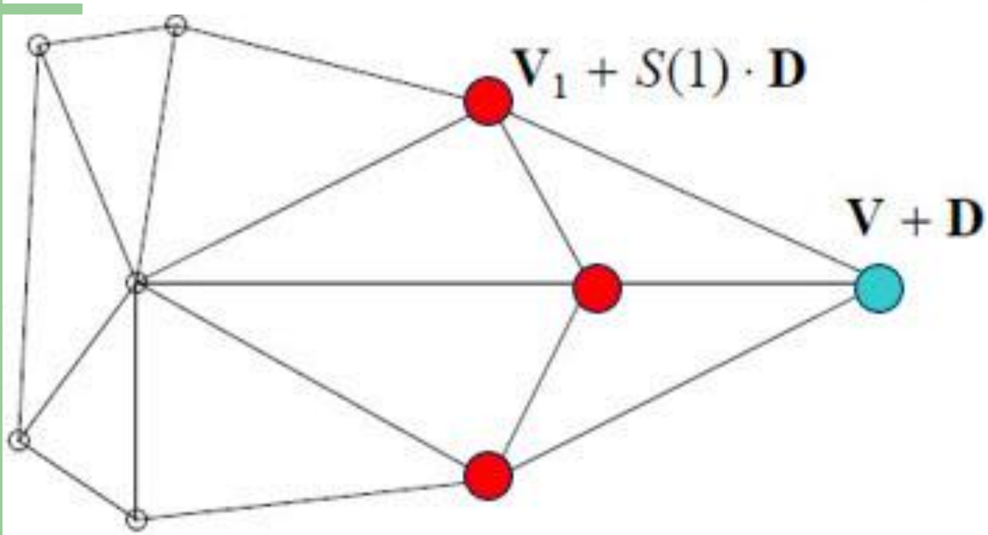
1. **Input correspondences at key feature points**
2. **Define a triangular mesh over the points**
   - Same mesh in both images!
   - Now we have triangle-to-triangle correspondences
3. **Warp each triangle separately from source to destination**

Or,

Warping an Object: picking and pulling



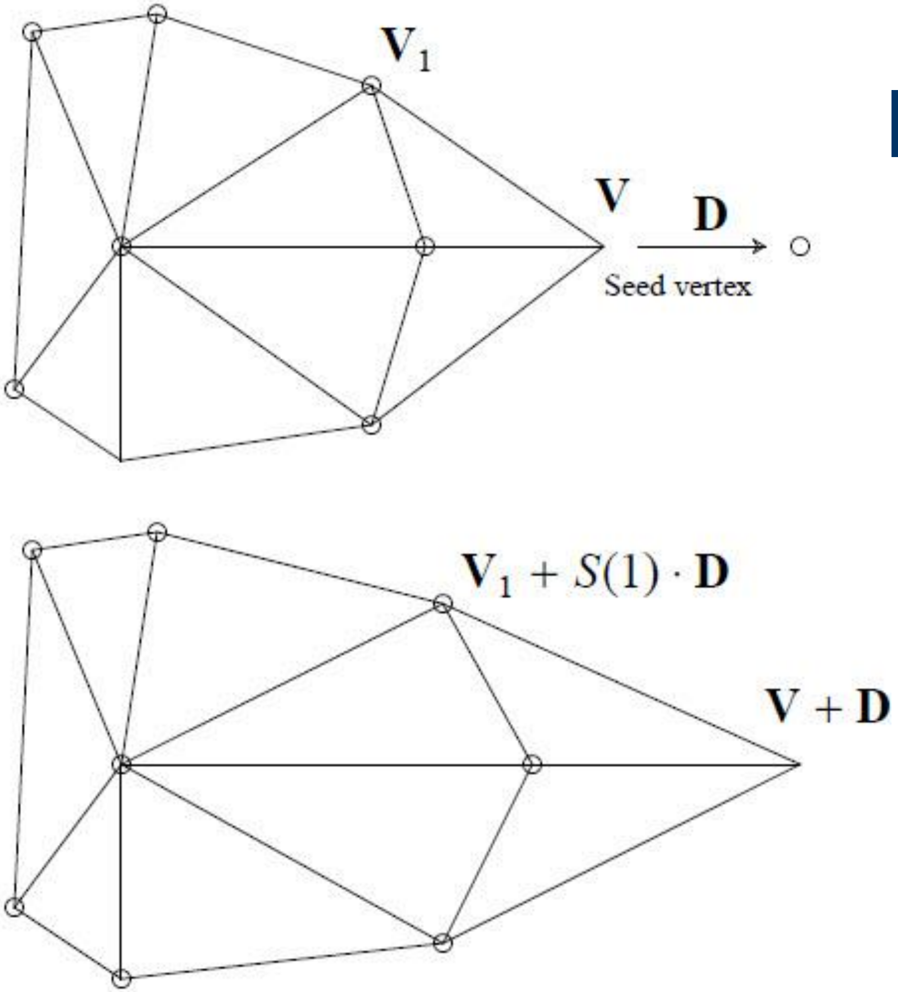
- displace the seed vertex (vertices) and propagate the displacement to adjacent vertices by attenuating initial displacement



D: displacement vector for seed vertex

n: range of propagation

Warping an Object: picking and pulling



For a vertex *i* units away from the seed vertex, the displacement is

$$S(i)\ \mathbf{D}$$

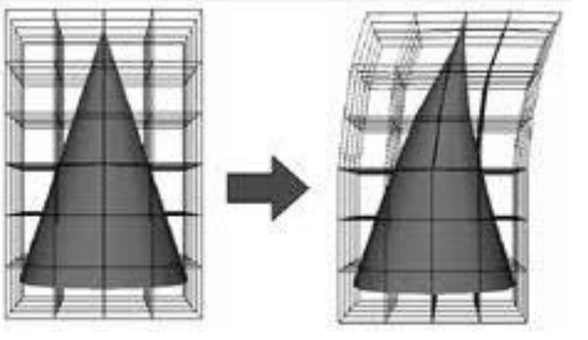
$$S(i) = 1.0 - \left(\frac{i}{n+1}\right)^k, \qquad k \geq 0$$

k = 1: linear attenuation

k > 1: create a more elastic impression

Deforming an embedding space (free-form deformation (FFD))

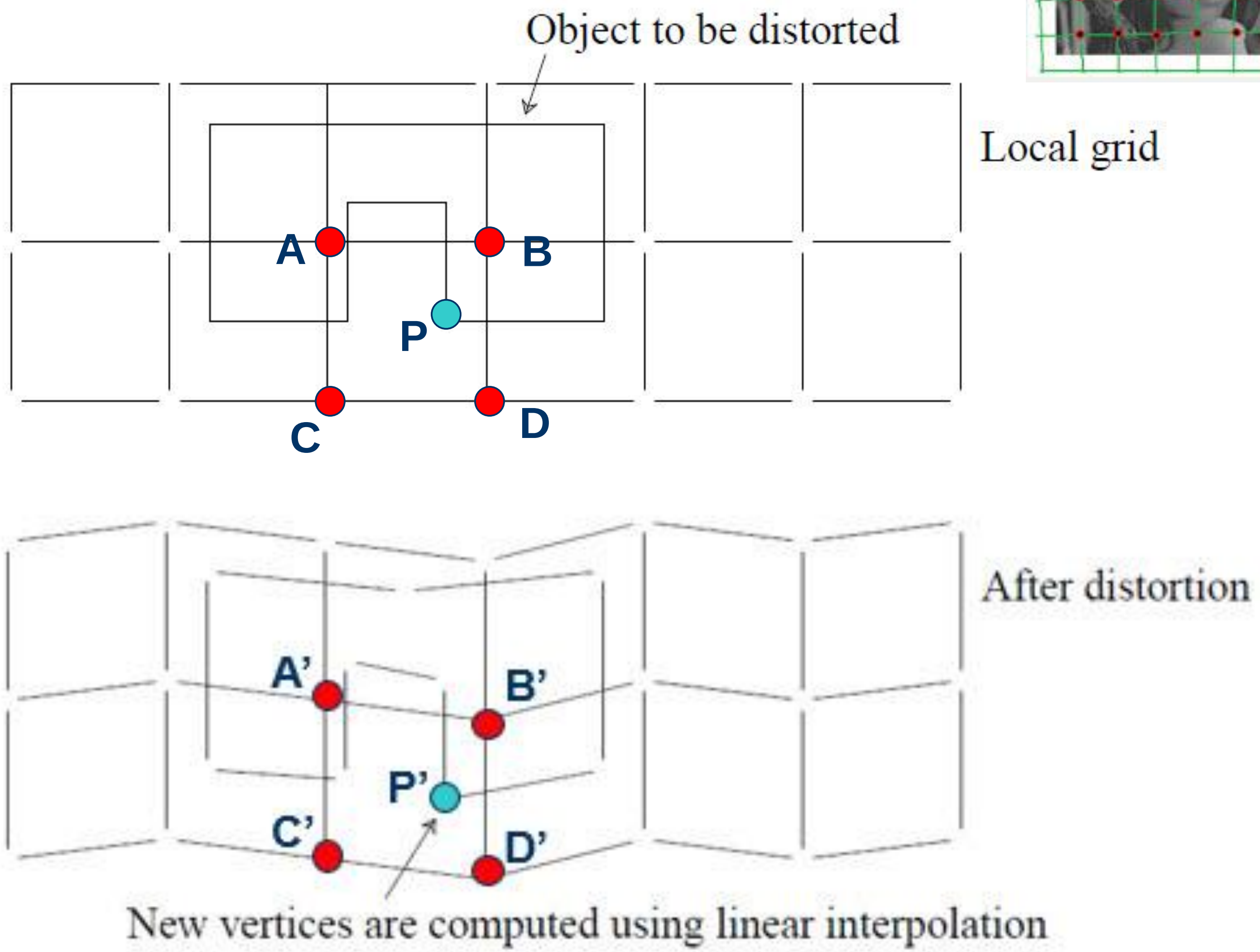
- include the object to be distorted in a local grid and then distort the local grid



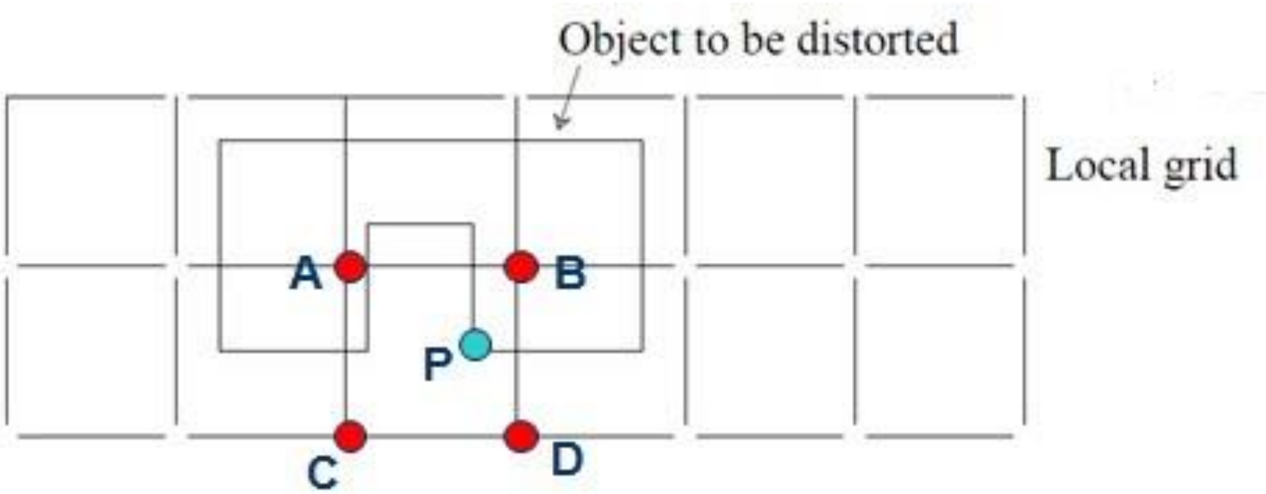
- easier to manipulate the local grid than to manipulate the vertices of the object directly
- more powerful than *affine transformations* because the distortion of the local grid can be nonlinear

$$\begin{aligned} f &: X \to Y \\ x &\to Mx + b \end{aligned}$$

2D grid deformation



2D grid deformation

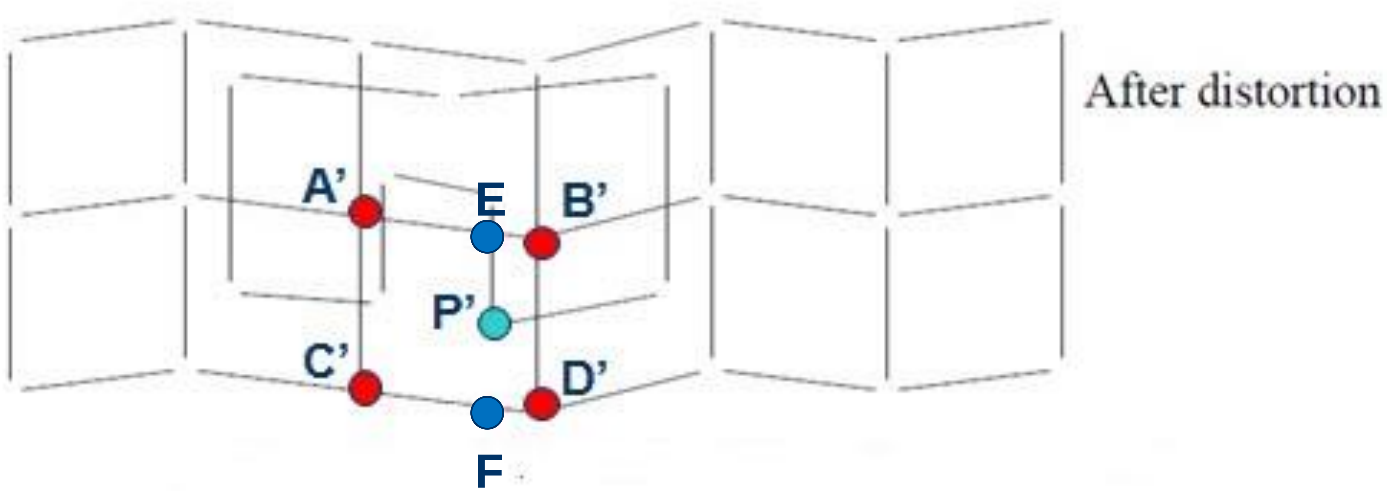


A = (5.0, 12.0) B = (6.0, 12.0)

C = (5.0, 11.0) D = (6.0, 11.0)

P = (5.7, 11.6)

2D grid deformation



E = 0.3 A' + 0.7 B'
F = 0.3 C' + 0.7 D'

P' = 0.4 F + 0.6 E

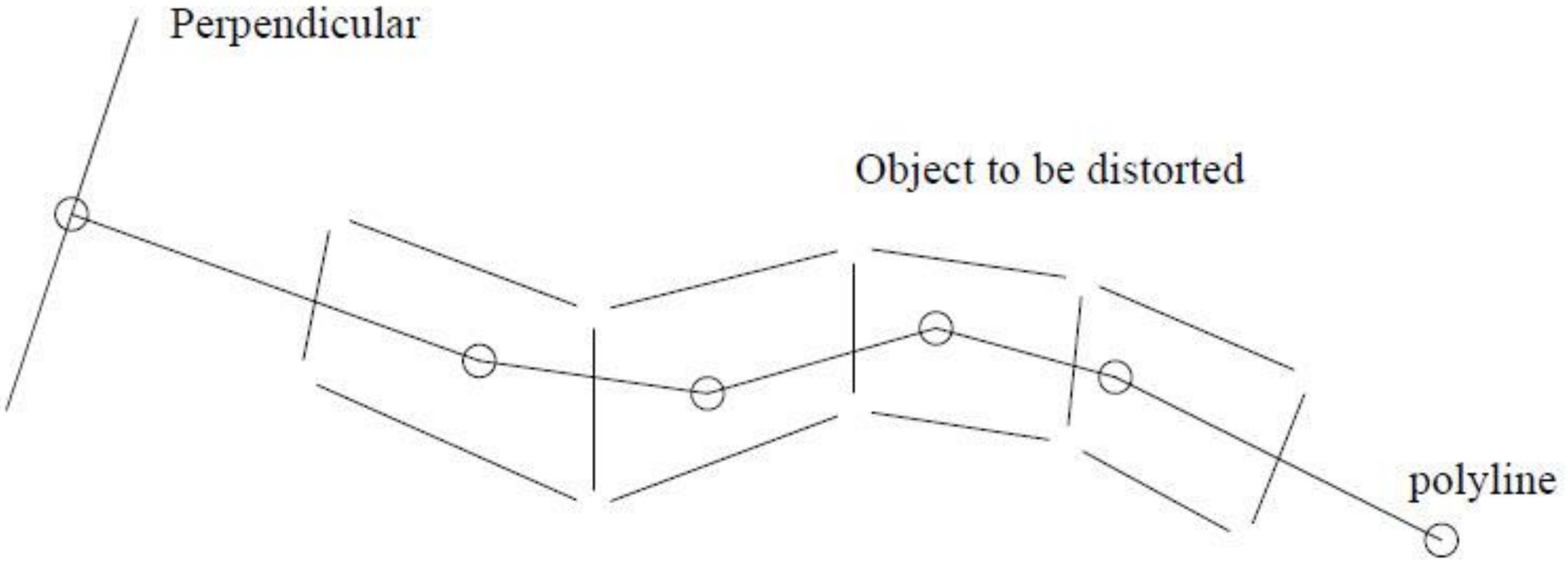
Polyline deformation (2D)

- draw a polyline thru the object to be distorted
- map object vertices to the polyline
- modify the polyline
- map object vertices to the same relative location of the polyline
- suitable for serpentine objects

of the shape of a snake

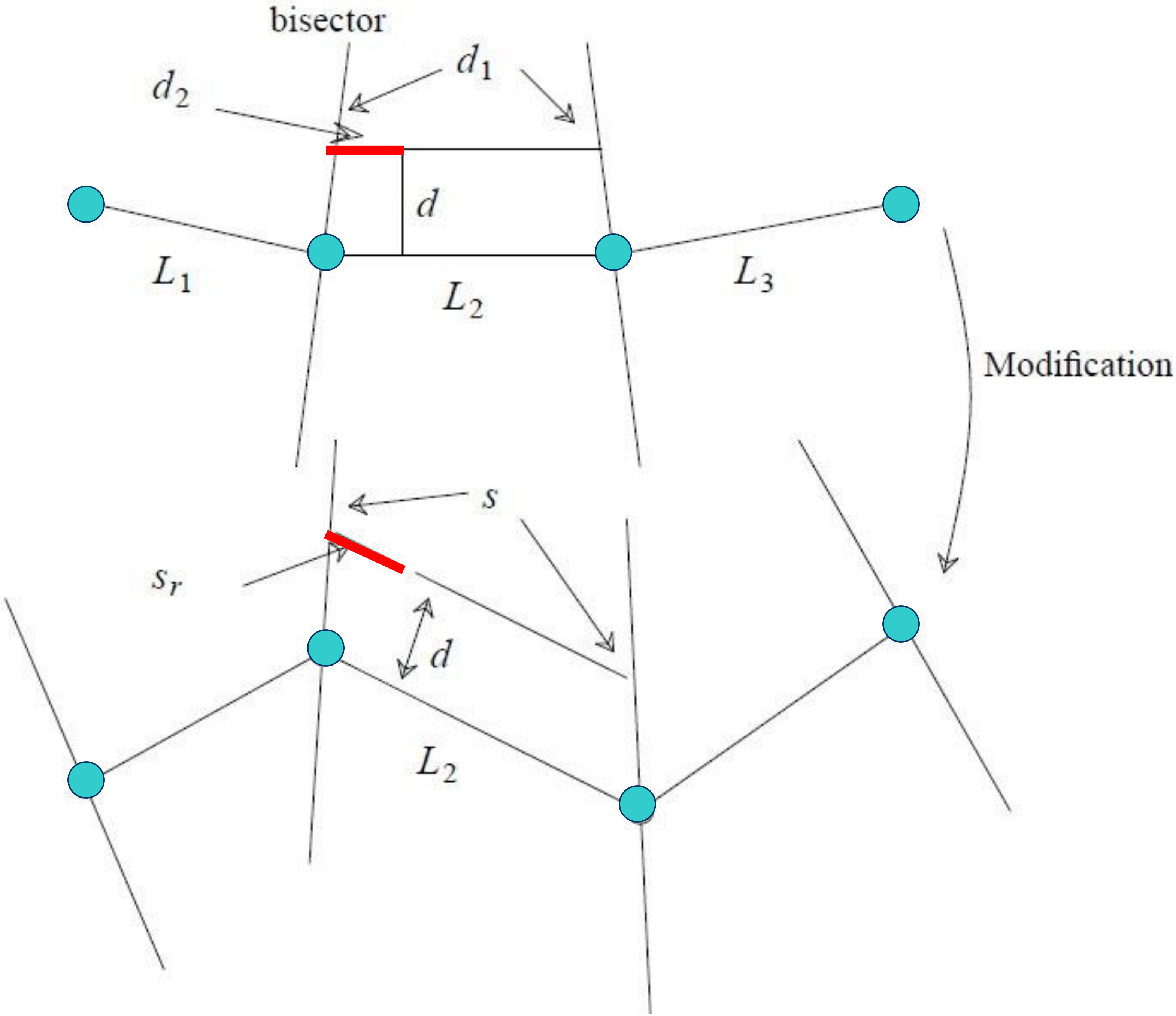
Polyline deformation (2D)

- draw a polyline thru the object to be distorted
- map object vertices to the polyline

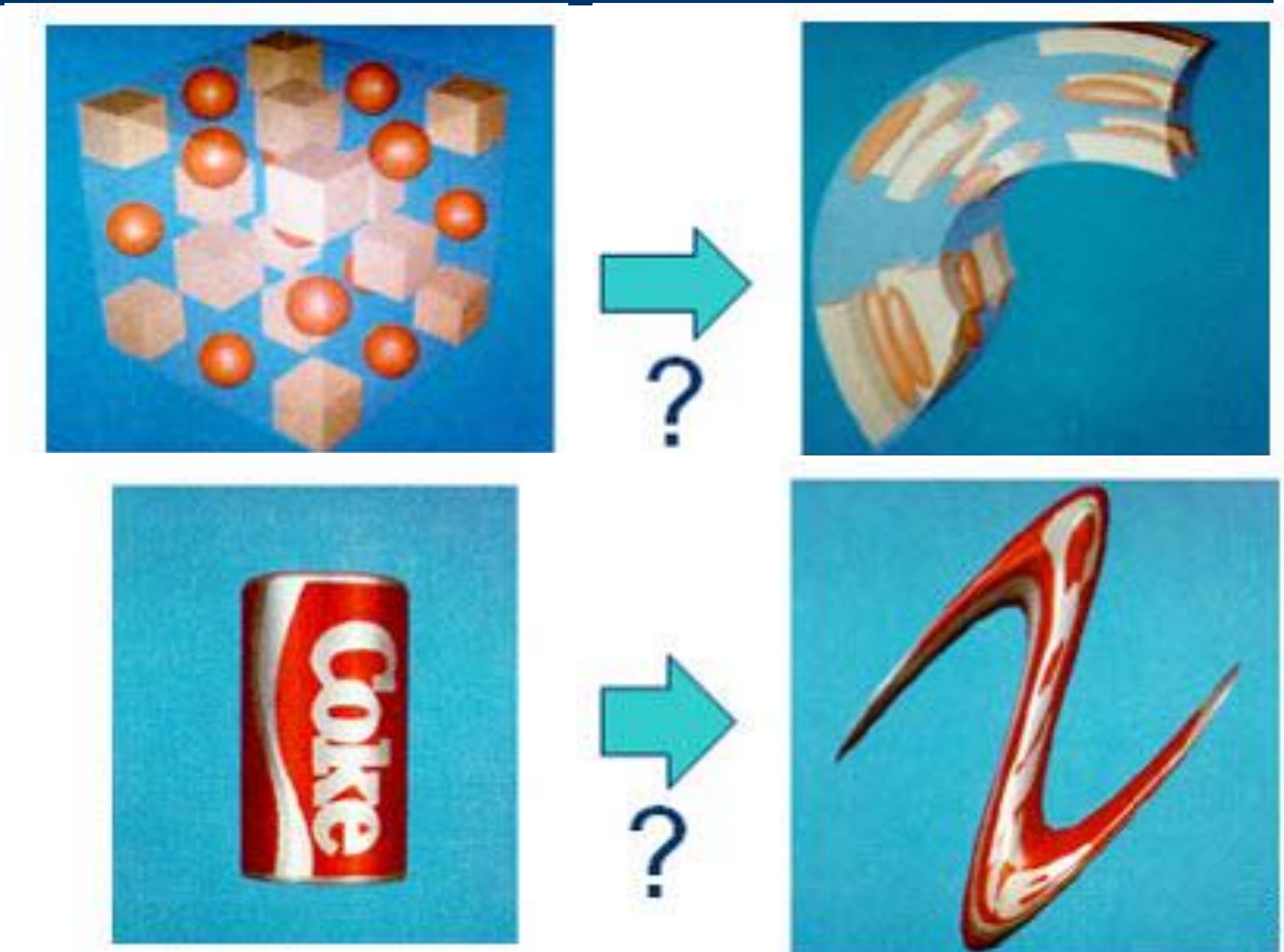


$$r = \frac{d_2}{d_1}$$

$$\frac{s_r}{s} = r$$



Free-Form Deformation (FFD): Sederberg

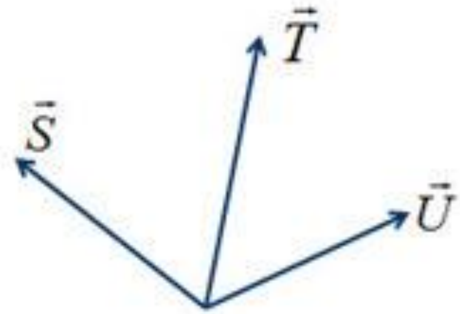


Free-Form Deformation (FFD):

- 3D extension of 2D grid deformation
- superimpose a localized coordinate grid over the object
- register vertices of the object to the grid (local coordinate system)
- manipulate the grid
- map object vertices back into the modified grid, then relocate them in global space

actually $\vec{S}, \vec{T}$ and $\vec{U}$ do not have to be unit vectors either.

If the local coordinate system is defined by $(\vec{S}, \vec{T}, \vec{U})$



$\vec{S}, \vec{T}, \vec{U}$: unit vectors but not necessarily orthogonal

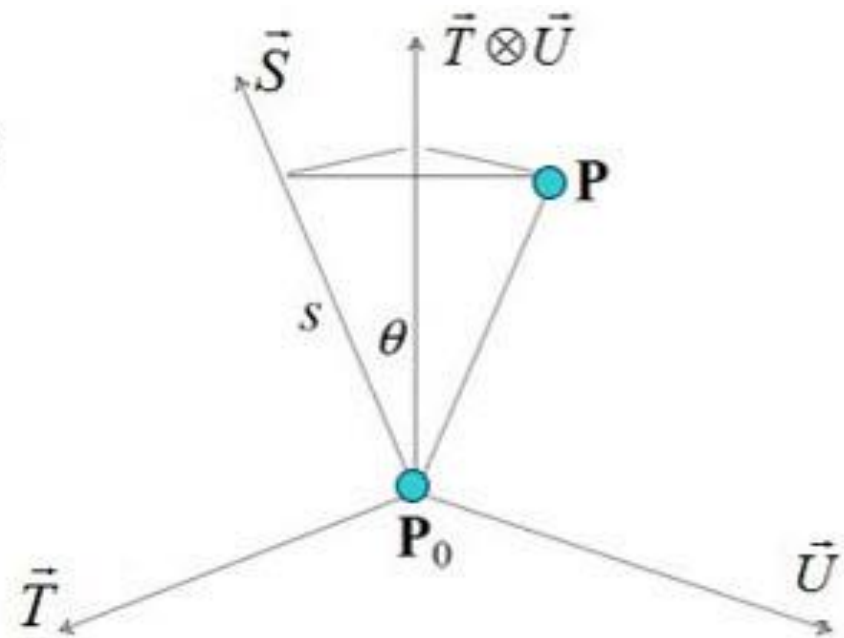
Relationship between local and global coordinate systems:

$$P = P_0 + s\vec{S} + t\vec{T} + u\vec{U}$$

P, P_0 : global

(s, t, u): local

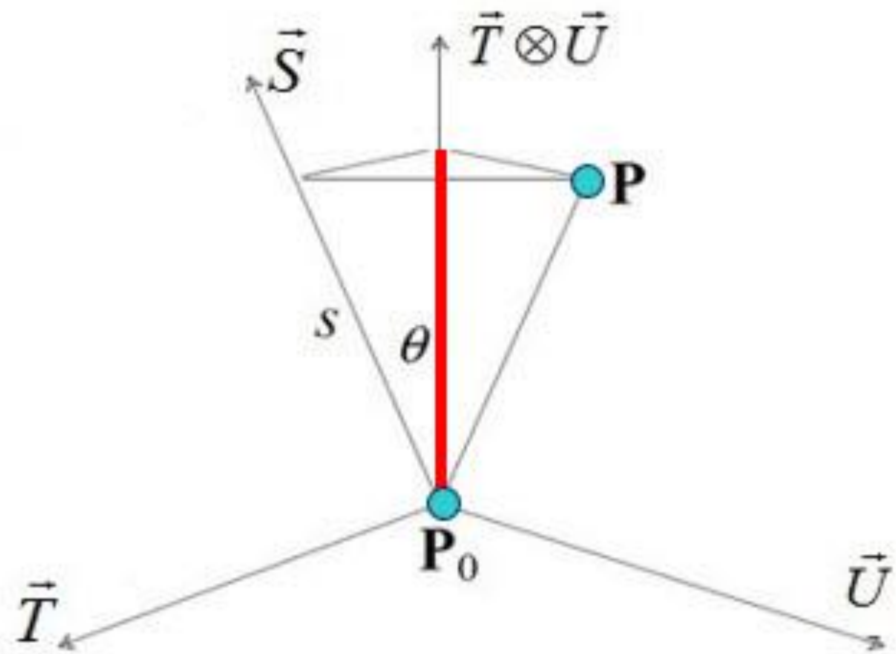
$\boxed{s = ? \quad t = ? \quad u = ?}$



$$P = P_0 + s\vec{S} + t\vec{T} + u\vec{U}$$

P, P_0 : global

$(s.t, u)$: local



$$s = \left(\vec{T} \otimes \vec{U}\right) \cdot \left(P - P_0\right) / ((\vec{T} \otimes \vec{U}) \cdot \vec{S})$$

$$t = (\vec{U} \otimes \vec{S}) \cdot (P - P_0) / ((\vec{U} \otimes \vec{S}) \cdot \vec{T}) \qquad (*)$$

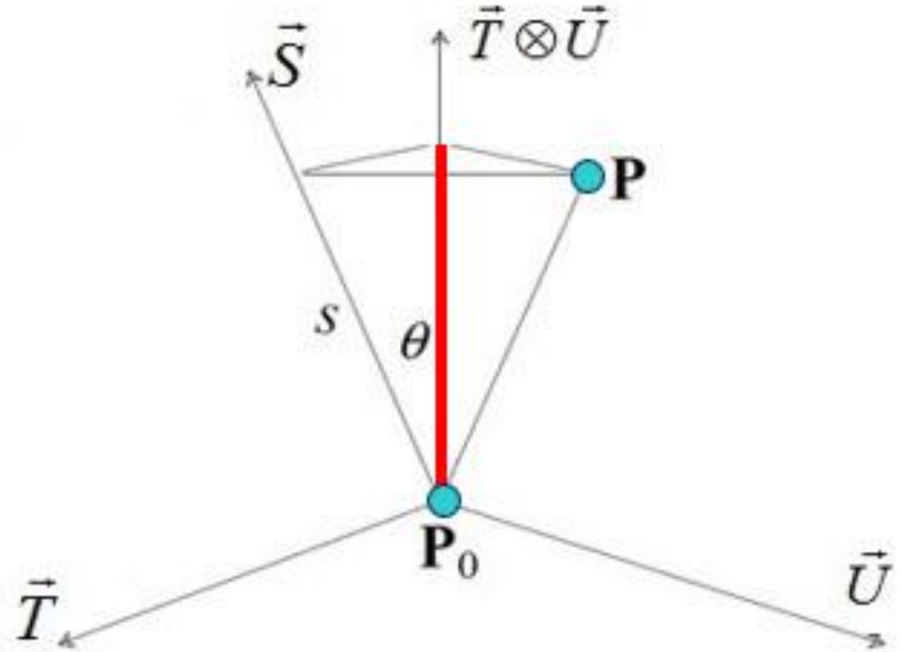
$$u = \left(\vec{S} \otimes \vec{T}\right) \cdot \left(P - P_0\right) / ((\vec{S} \otimes \vec{T}) \cdot \vec{U})$$

Why?

$$P = P_0 + s\vec{S} + t\vec{T} + u\vec{U}$$

P, P_0 : global

$(s. t, u)$: local



Here is why:

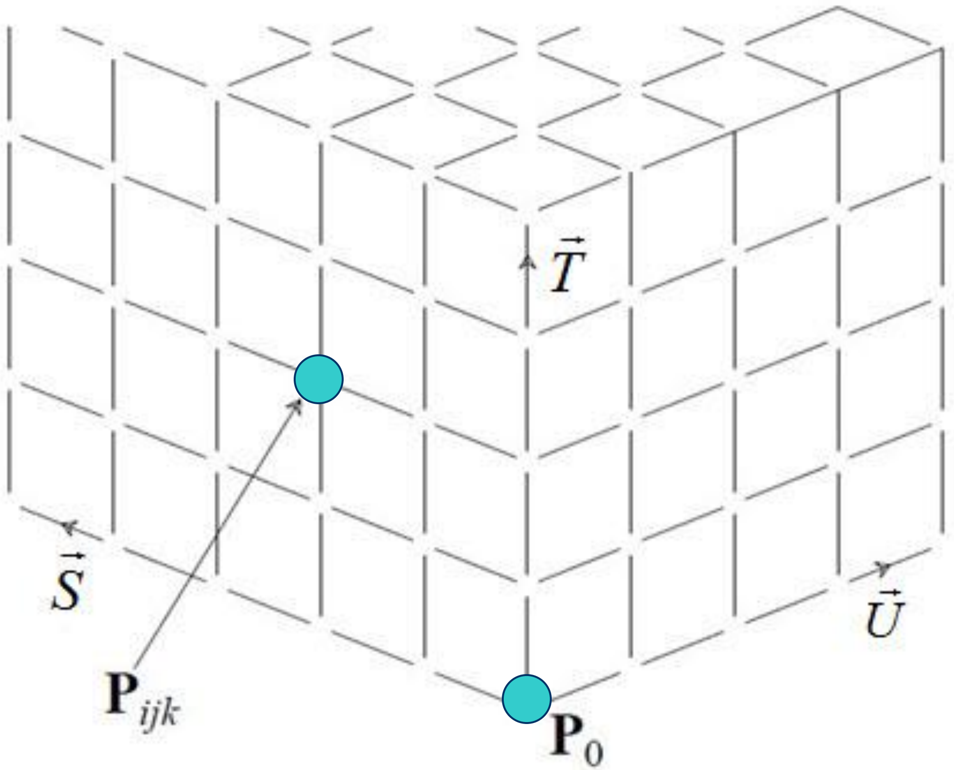
Since $\vec{T} \otimes \vec{U} \perp \vec{T}$ and $\vec{T} \otimes \vec{U} \perp \vec{U}$

we have

$$(P - P_0) \cdot (\vec{T} \otimes \vec{U}) = \left(s\vec{S} + t\vec{T} + u\vec{U}\right) \cdot (\vec{T} \otimes \vec{U}) = s\vec{S} \cdot (\vec{T} \otimes \vec{U})$$

Hence $s = (P - P_0) \cdot (\vec{T} \otimes \vec{U}) / (\vec{S} \cdot (\vec{T} \otimes \vec{U}))$

Creation of the localized coordinate grid

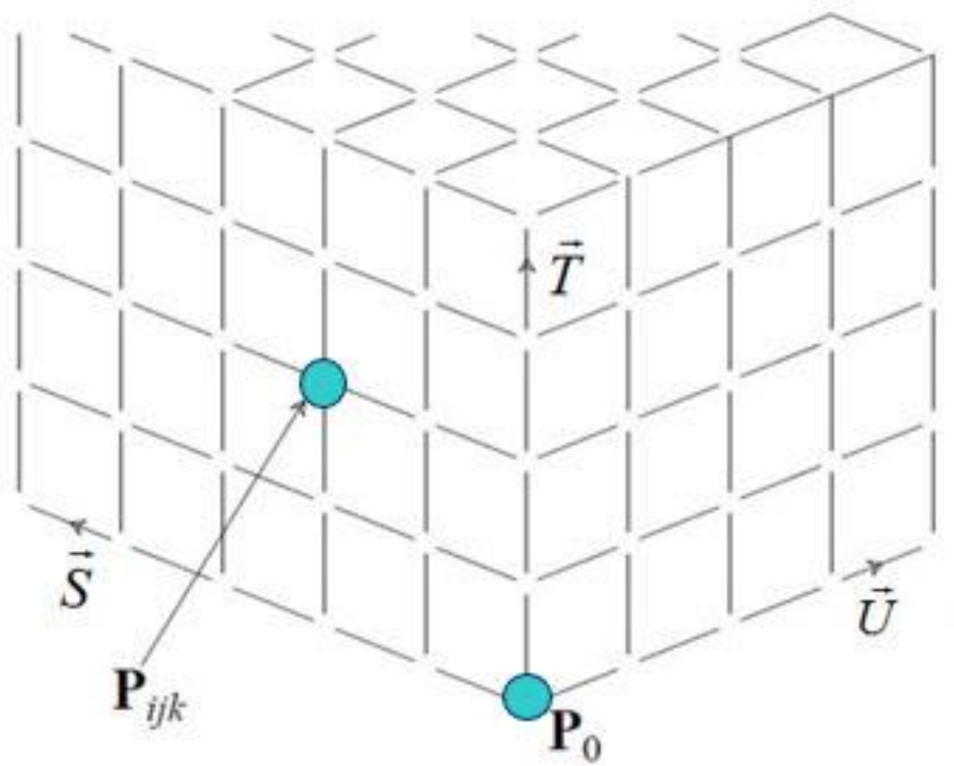


$$P_{ijk} = P_0 + \frac{i}{l}\vec{S} + \frac{j}{m}\vec{T} + \frac{k}{n}\vec{U}$$

l : # of control pts in $\vec{S}$ *direction*

m : # of control pts in $\vec{T}$ *direction*

n : # of control pts in $\vec{U}$ *direction*



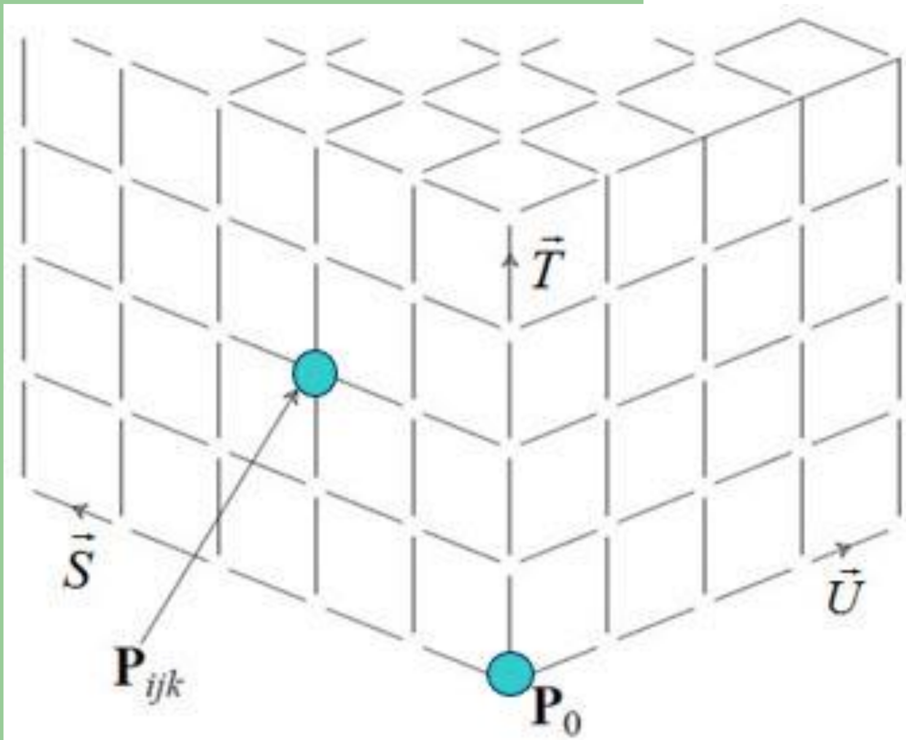
Animator adjusts the locations of the control points P_{ijk} to deform the object

Deformed position of a vertex of the object is determined through a *trivariate Bezier interpolation* process:

$$\mathbf{P}(s,t,u) = \sum_{i=0}^{l} \binom{l}{k} (1-s)^{l-i} s^i \cdot$$

$$\left\{ \sum_{j=0}^{m} \binom{m}{j} (1-t)^{m-j} t^j \left[\sum_{k=0}^{n} \binom{n}{k} (1-u)^{n-k} u^k \mathbf{P}_{ijk} \right] \right\}$$

$$0 \le s,\, t,\, u \le 1$$



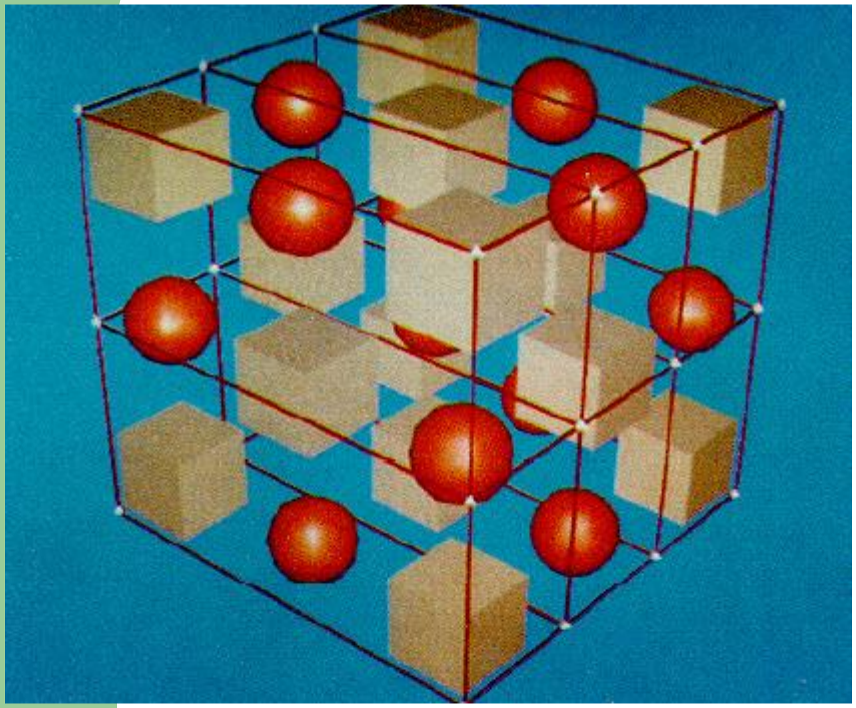
The deformation is specified by moving the $P_{i,j,k}$ from their un-displaced, latticial positions.

The deformed position X_{ffd} of an arbitrary point X is found by first computing its (s, t, u) coordinates from equation (*), and then evaluating the vector valued tri-variate Bernstein polynomial:

$$\mathbf{X}_{\mathbf{ffd}} = \sum_{i=0}^{l} \binom{l}{k} (1-s)^{l-i} \; s^{i} \cdot$$

$$\left\{ \sum_{j=0}^{m} \binom{m}{j} (1-t)^{m-j} \; t^{j} \left[\sum_{k=0}^{n} \binom{n}{k} (1-u)^{n-k} \; u^{k} \mathbf{P}_{ijk} \right] \right\}$$

new location



FFDs can be composed
- sequentially vs hierarchically

can be sub-dimensional
or muli-dimensional

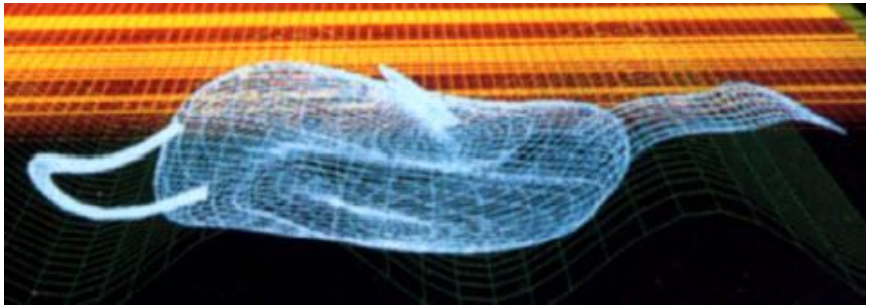
Detail elements can be added in stages

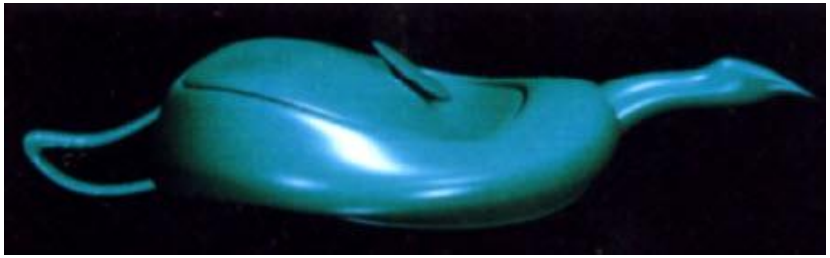
usually in the same
direction

Allows the user to work at various levels of detail

Free form deformation through control of parametric surfaces

Free form deformation through control of parametric surfaces





End of Interpolation IV