CS633 3D Computer Animation
Homework Assignment 1 (40 points)
Due: 2/4/2014

You need to do some study and research on Questions 1 and 2.

1. What is the difference between *computer graphics* and *computer animation*? Actually in computer graphics sometimes we also deal with motions. What is the main difference between the motions dealt with in computer graphics and those dealt with in computer animation? (5 points)

2. What is the difference between *playback rate* and *sampling rate*? Why sampling the scene every sixtieth of a second and then playing back the frames at thirty frames per second would produce *smoother motion*? (5 points)

3. A quaternion q can be defined as follows

$$q \equiv w + xi + yj + zk$$

where w, x, y, z are real numbers, $i^2 = j^2 = k^2 = -1$, $ij = -ji = k$, $jk = -kj = i$ and $ki = -ik = j$. Prove that if the quaternion q is defined as follows

$$q \equiv [s, \mathbf{v}]$$

where s is a scalar and $\mathbf{v}$ is a 3-component vector, with s playing the role of w and $\mathbf{v}$ playing the role of $xi+yj+zk$, then the product of two quaternions $q_1 = [s_1, \mathbf{v}_1]$ and $q_2 = [s_2, \mathbf{v}_2]$ should be defined as follows:

$$q_1 * q_2 \equiv [s_1 s_2 - \mathbf{v}_1 \cdot \mathbf{v}_2, \quad s_1\mathbf{v}_2 + s_2\mathbf{v}_1 + \mathbf{v}_1 \otimes \mathbf{v}_2]$$

where $\mathbf{v}_1 \cdot \mathbf{v}_2$ is the inner product of $\mathbf{v}_1$ and $\mathbf{v}_2$, and $\mathbf{v}_1 \otimes \mathbf{v}_2$ is the cross product of $\mathbf{v}_1$ and $\mathbf{v}_2$. (5 points)

4. The inverse of a quaternion q is defined as

$$q^{-1} \equiv \frac{q^*}{qq^*}$$

Define $(q^{-1})^{-1}$. Is $(q^{-1})^{-1} = q$? (5 points)

5. Given two quaternions q_1 and q_2, prove that

$$| \, q_1 * q_2 \, | = |q_1| \, |q_2|$$

(5 points)

6. Is the product operation of quaternions commutative? Why or Why not? (5 points)

7. If p and q are two quaternions and p^{-1} and q^{-1} are their inverses, prove that

$$(p * q)^{-1} = q^{-1} * p^{-1}$$

(5 points)

8. Rotation of a point $\mathbf{P}$ by an angle θ about a rotation axis $\mathbf{u}$ (a unit vector) is defined as follows:

$$q * [0, \mathbf{P}] * q^{-1}$$

where $q = [cos(\theta/2), sin(\theta/2)\ \mathbf{u}]$. If we perform two consecutive rotations about the same rotation axis, $\mathbf{u}$, how should the result be defined? i.e., if we rotate $\mathbf{P}$ about $\mathbf{u}$ for θ_1 first, then for θ_2, how should the result be defined in terms of quaternion multiplication? (5 points)