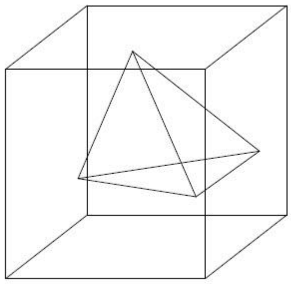
CS 633 3D Computer Animation
Solution Set - HW 4 (40 points)

Due: 3/15/2018

1. Given two *star-shaped* polyhedra, how would you modify their vertex-edge-face structures (boundary representations) so that they would have the same topology (so that we can perform interpolation on corresponding vertices to generate intermediate shapes in the process of changing one object into the other)? Use the following cube and the enclosed pyramid to explain your concept. Remember, at the end, the modified boundary representations should have the same vertex-edge-face structure. (10 points)



Sol.

(1) First, find a point that is inside both objects (such as the centroid of the enclosed pyramid), call that point *O*.

(2) For each vertex of the pyramid, generate a ray from *O* to that vertex and find its intersection point with the cube. Do the same thing for vertices of the cube.

(3) If a face of the pyramid does not contain any of the new intersection points, keep that face. Otherwise, use the intersection points to break the face into sub-faces. For instance, if the face contains one intersection point, then use that intersection point and vertices of the face to form three new sub-faces. If the face contains two intersection points, first use one of the intersection points to break the face into three sub-faces. One of the sub-faces would contain the second intersection point as an interior point. Then use the second intersection point to break that sub-face into three sub-sub-faces.

(4) At this point, the new vertex-edge-face structures of the cube and the pyramid would be the same.

2. Question: can any genus-0 polyhedron be mapped to a sphere? If your answer is YES, prove it (must be precise). Otherwise, use an example to explain why this is not possible. (10 points)

Sol.

The answer is YES. The following expansion algorithm can be used to construct such a mapping.

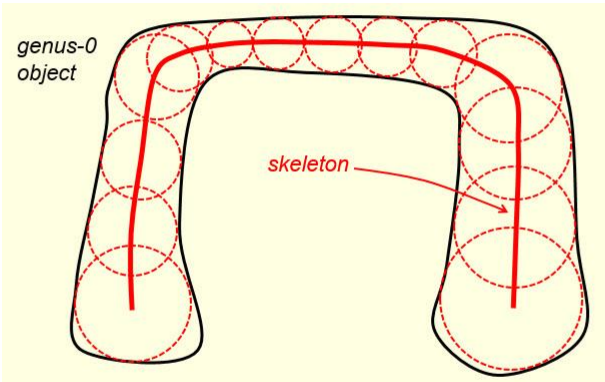
(1) First construct a skeleton of the given genus-0 polyhedron. (What is a skeleton of a polyhedron? See the following figure for a 2D example)

(2) Set *r* to be the largest radius in constructing the skeleton

(3) Repeat the following process until the polyhedron is a good approximation of a sphere:

(a) Traverse the skeleton of the polyhedron with a sphere of radius *r*. For each point of the skeleton, if *r* is bigger than the radius of the sphere for that point, expand the polyhedron (how?) around that point until radius of the sphere for that point is $\geq r$.

(b) Construct a skeleton for the new polyhedron, increment *r* by a pre-set step size, then repeat the above step.



3. To generate an intermediate image in the *coordinate grid* based *morphing* process, one has to generate an *intermediate grid* first. The source image and the destination image are then both warped according to the intermediate grid in a two-pass process. The warped images are then cross-dissolved to form the intermediate image. Question: can the two-pass warping process be merged into a one-pass process? Why or why not? (10 points)

Sol.

The yes is YES.

The two-pass process consists of an *x-warping* and a *y-warping*. The *x-warping* is a one-dimensional re-sampling process:

$$f: (x, y) \rightarrow (f(x) , y)$$

and the *y-warping* is also a one-dimensional re-sampling process

$$g: (x, y) \rightarrow (x , g(y))$$

Therefore, the *x*-*warping* and the *y*-*warping* processes can be combined into a single process as

$$F: (x, y) \rightarrow (\, f(x)\, ,\, g(y))$$

But this is not preferred because the result is not intuitively clear to the user.

4. If *multiple* feature lines are used in the *feature* based *morphing* process, a relative weight has to be computed for each feature line. The source image locations computed for all the feature lines are then averaged to get a final source image location for the corresponding pixel in the intermediate image. Does this approach reduce to the technique shown on slide 40 of notes: Interpolation-based Animation II, when there is only one feature line given? (10 points)

Sol.

The answer is NO. Because even if only one feature line is used, one still has to compute a weight, and the weight is not one and, consequently, does not reduce to the case on slide 40.