CS633 3D Computer Animation
Homework Assignment 2 (40 points)
Due: 2/15/2018

1. Let q be a rotation quaternion, i.e., q is a quaternion of the following form

$$q = [\cos(\theta), \sin(\theta)(x, y, z)]$$

with (x, y, z) being a unit vector. The question you need to answer here is: Does $-q$ represent the same rotation? Justify your answer. (5 points)

2. Each rotation quaternion is a unit quaternion (a quaternion with unit length). Is a unit quaternion also a rotation quaternion? Justify your answer. (5 points)

3. I have shown you in class how to find the control points of a uniform cubic B-spline curve that interpolates six given data points $\mathbf{D}_0$, $\mathbf{D}_1$, $\mathbf{D}_2$, $\mathbf{D}_3$, $\mathbf{D}_4$ and $\mathbf{D}_5$ (slide 7 of notes: Interpolating Values I). This curve has five segments. So, it needs eight control points $\mathbf{P}_0$, $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$, $\mathbf{P}_5$, $\mathbf{P}_6$ and $\mathbf{P}_7$. To find these control points, you need to solve a system of four equations to find $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$ and $\mathbf{P}_5$ first, and then compute $\mathbf{P}_0$ and , $\mathbf{P}_7$. You don't have to compute $\mathbf{P}_1$ and $\mathbf{P}_6$ because they are equal to $\mathbf{D}_0$ and $\mathbf{D}_5$, respectively. Now, here is the question: to build a uniform cubic B-sline curve to interpolate 10 data points: $\mathbf{D}_0$, $\mathbf{D}_1$, ..., $\mathbf{D}_8$ and $\mathbf{D}_9$, how many segments should it have? and how many control points should this curve have? (2.5 points)

4. To build a uniform cubic B-spline curve to interpolate 10 data points: $\mathbf{D}_0$, $\mathbf{D}_1$, ..., $\mathbf{D}_8$ and $\mathbf{D}_9$, we also need to solve a system of equations for some of the control points. How big is this system (i.e., how many equations are there in this system)? Which control points (using the notations of Question 3) will be computed from this system? (2.5 points)

5. Here is a more specific question. What is the exact form of the system of equations to be solved in Question 4? Give your answer in matrix form. (5 points)

6. If we use the approach covered on slides 8-19 of Interpolating Values I, then what system of equations should we solve to get the control points, in matrix form? (5 points)

7. If we want the user to have an impression that the object is moving in constant speed, we need to parametrize the path curve by arc length. For a B-spline curve, how should the re-parametrization process be done? Note that a B-spline curve usually has several segments and each segment is defined differently. (5 points)

8. Prove the third equation on slide 30 of notes: Interpolating Values II, i.e.,

$$s_3(t) = \frac{v_0 t_1}{2} + v_0(t_2 - t_1) + [v_0 - \frac{v_0(t - t_2)}{2(1 - t_2)}](t - t_2)$$

when $t_2 < t < 1$. To prove the above equation, you need to show the following equation first. (5 points)

$$v_3(t) = -\frac{v_0}{1 - t_2}(t - t_2) + v_0.$$

9. There are two approaches to define a *spherical linear interpolation* (see slides 4 and 8 of notes: Interpolating Values III for the definitions of this term) between two unit quaternions. In the second approach, *slerp*(q_1, q_2; u) is defined as follows:

$$slerp(q_1, q_2; u) = \frac{\sin((1 - u)\theta)}{\sin\theta} q_1 + \frac{\sin(u\theta)}{\sin\theta} q_2$$

Prove the second approach generates the same curve as the first approach. (5 points)