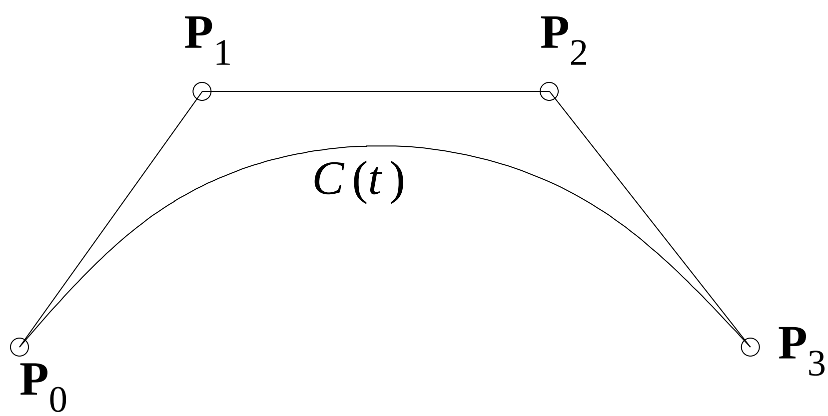
3.1.1 Bezier Curve Segments of Degree 3

$$\mathbf{C}(t) = (1-t)^3\mathbf{P}_0 + 3t(1-t)^2\mathbf{P}_1 + 3t^2(1-t)\mathbf{P}_2 + t^3\mathbf{P}_3$$

$$0 \le t \le 1$$

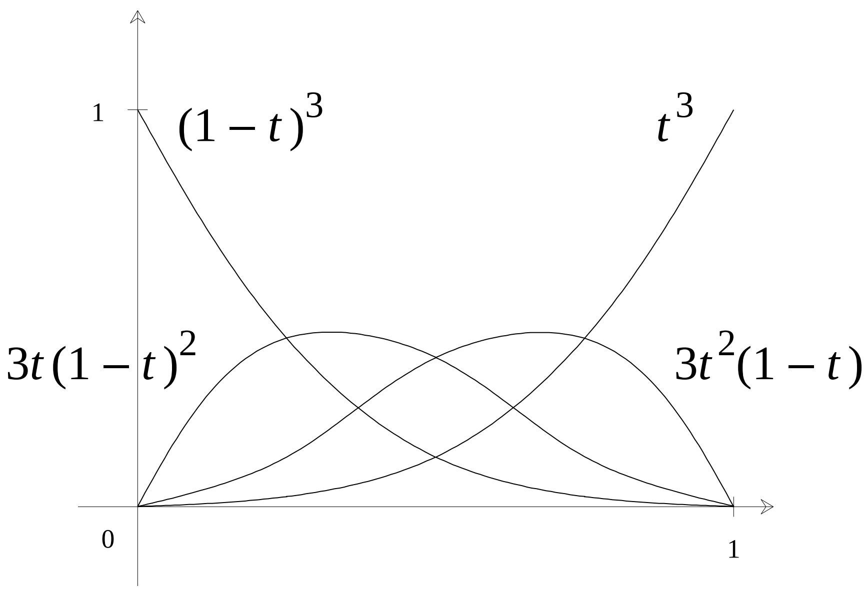


Matrix form:

$$\mathbf{C}(t) = [1, t, t^2, t^3] \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{P}_0 \\ \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix}$$

$$= \mathbf{T} \cdot \mathbf{M}_b \cdot \mathbf{G}$$

- $\mathbf{P}_i = (x_i, y_i)$ are called **control points**
- The polygon $\mathbf{P}_0\mathbf{P}_1\mathbf{P}_2\mathbf{P}_3$ is called the **control polygon**
- The weights $(1-t)^3$, $3t(1-t)^2$, $3t^2(1-t)$, and t^3 are called **blending functions**



Notes:

- Blending functions are always non-negative
- Blending functions always sum to 1

- $C(0) = \mathbf{P}_0$; $C(1) = \mathbf{P}_3$

  (A Bezier curve always starts at $\mathbf{P}_0$ and ends at $\mathbf{P}_3$.

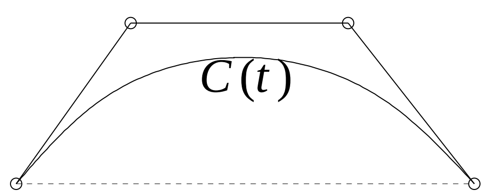
- $C'(0) = 3(\mathbf{P}_1 - \mathbf{P}_0)$; $C'(1) = 3(\mathbf{P}_3 - \mathbf{P}_2)$

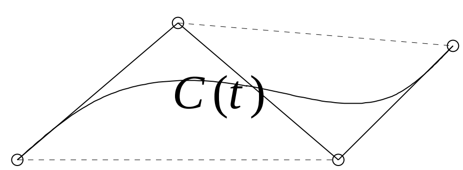
  (A Bezier curve is tangent to the control polygon at the endpoints)

- $C''(0) = 6(\mathbf{P}_2 - 2\mathbf{P}_1 + \mathbf{P}_0)$; $C''(1) = 6(\mathbf{P}_3 - 2\mathbf{P}_2 + \mathbf{P}_1)$

- Bezier curve segments satisfy **convex hull property**

  i.e., a Bezier curve segment is always contained in the **convex hull** of its control points



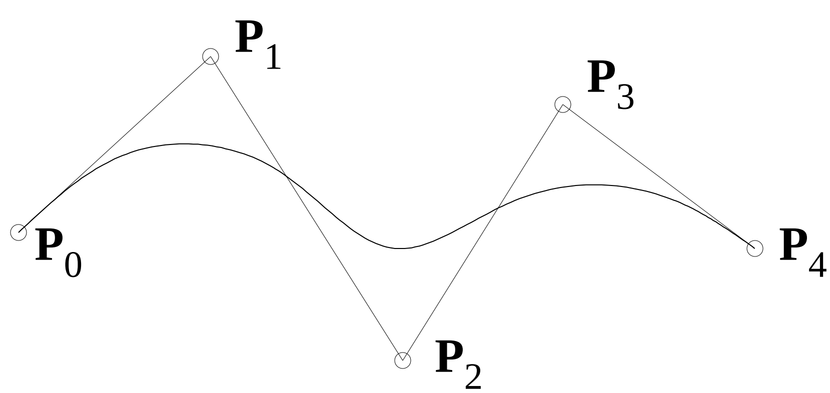


- Bezier curves have intuitive appeal for interactive users

3.1.2 General Bezier Curves

$$C(t) = \sum_{i=0}^{n} B_{i,n}(t)\,\mathbf{P}_i = \sum_{i=0}^{n} \binom{n}{i} t^i (1-t)^{n-i}\,\mathbf{P}_i,$$

where $0 \le t \le 1$ and $\binom{n}{i} \equiv \frac{n!}{i!\,(n-i)!}$. $B_{i,n}(t)$ are again called **blending functions** and $\mathbf{P}_i$ **control points.**



- All the properties mentioned on pages 54 and 55 hold for general Bezier curves.

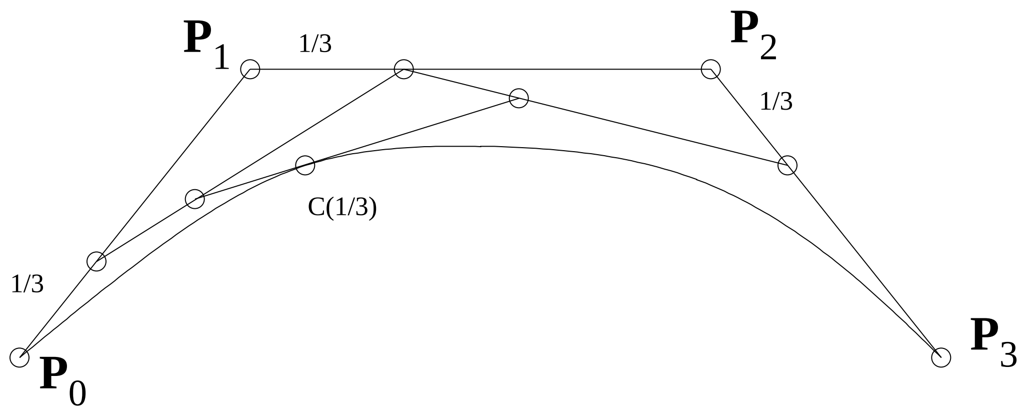
<u>A</u> <u>recurrance</u> <u>relation:</u>

$$C(t) = (1-t)\left(\sum_{i=0}^{n-1} B_{i,n-1}(t)\,\mathbf{P}_i\right) + t\left(\sum_{i=0}^{n-1} B_{i,n-1}(t)\,\mathbf{P}_{i+1}\right)$$

$$= (1-t)\cdot\left[\sum_{i=0}^{n-1} \binom{n-1}{i} t^i\,(1-t)^{n-1-i}\,\mathbf{P}_i\right]$$

$$+\,t\cdot\left[\sum_{i=0}^{n-1} \binom{n-1}{i} t^i\,(1-t)^{n-1-i}\,\mathbf{P}_{i+1}\right]$$

- Curve computation

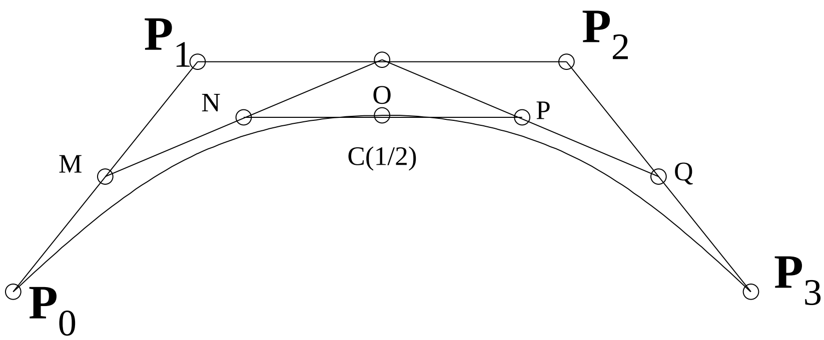


If degree = 3 then

$$C(\frac{1}{3}) = \frac{2}{3}\left[\frac{2}{3}\left(\frac{2}{3}\mathbf{P}_0 + \frac{1}{3}\mathbf{P}_1\right) + \frac{1}{3}\left(\frac{2}{3}\mathbf{P}_1 + \frac{1}{3}\mathbf{P}_2\right)\right]$$

$$+\frac{1}{3}\left[\frac{2}{3}\left(\frac{2}{3}\mathbf{P}_1 + \frac{1}{3}\mathbf{P}_2\right) + \frac{1}{3}\left(\frac{2}{3}\mathbf{P}_2 + \frac{1}{3}\mathbf{P}_3\right)\right]$$

- Midpoint Curve Subdivision



$\mathbf{P}_0, M, N, O$ are control points of $C(t)$, $0 \le t \le 1/2$, and $O, P, Q, \mathbf{P}_3$ are control points of $C(t)$, $1/2 \le t \le 1$.

- Recursively subdivide the control polygons at the midpoints, we can divide the curve into many small segments, each with its own control points.

- These control points, when connected, form a good linear approximation of the curve $\mathbf{C}(t)$. (This linear approximation is usually used to to find the intersection points of two Bezier curves)