Solution to CS375 Homework Assignment 2 (40 points)
Due Date: February 10, 2016

1. Find a regular expression to describe the given language: \{ aa, ab, ac \} ( 5 points )
   **Solution:**
   The regular expression that describes the given language is: \( a(a+b+c) \)

2. Find a regular expression for the given language over the alphabet \{a,b\}: strings with an odd number of a’s. ( 5 points )
   **Solution:**
   The regular expression for the language of strings over the alphabet \{a, b\} with an odd number of a’s:
   \( b^*a(b^*ab^*a)^*b^* \)

3. Simplify the given regular expression: \( aa(b^*+a) + a(ab^*+aa) \) (5 points)
   **Solution:** two approaches
   \[
   \begin{align*}
   aa(b^*+a) + a(ab^*+aa) &= aab^* + aaa + aab^* + aaa \\
   &= aab^* + aaa \quad R + R = R \\
   &= aa(b^*a) \\
   \text{or} \\
   aa(b^*+a) + a(ab^*+aa) &= aa(b^* + a) + aa(b^* + a) \\
   &= aa(b^*a) \\
   
   \end{align*}
   \]

4. Prove the following equality of regular expression: \( a^*(b + ab^*) = b + aa*b* \) ( 5 points )
   **Solution:**
   \[
   \begin{align*}
   a^*(b + ab^*) &= (\Lambda + aa^*)(b + ab^*) \\
   &= b + ab^* + aa*b + aa*ab^* \\
   &= b + (\Lambda + aa^*)ab^* + aa*b \\
   &= b + a*ab^* + aa*b \quad \Lambda + RR^* = R^* \\
   &= b + aa*b^* + aa*b \\
   &= b + aa*(b^* + b) \quad R^*R = RR^* \\
   &= b + aa*b^* \quad R^* + R = R^* \\
   \text{Hence proved.}
   \end{align*}
   \]

5. Prove the property of regular expression: \( R^* = (\Lambda + R) R^* \) ( 10 points )
   **Solution:** Two approaches
   Since \( L((\Lambda + R) R^*) = L(R^* + RR^*) = L(R^*) \cup L(R) L(R)^* = L(R)^* \cup L(R)^+ = L(R)^* \)
   so we have \( (\Lambda + R) R^* = R^* \)
(Λ + R) R* = (Λ + R) *
We need to prove that L(Λ + R) * = (L(Λ) ∪ L(R))* = L(R*)
Λ ∈ L(R) is trivial, so we assume that Λ ∉ L(R)
If x ∈ L(R)*, then x ∉ L(R)n for some n ∈ N. But then x ∈ (L(Λ) ∪ L(R))n.
So L(R)* ⊆ (L(Λ) ∪ L(R))n ⊆ (L(Λ) ∪ L(R))*.
On the other hand, if x ∈ (L(Λ) ∪ L(R))* then x ∉ (L(Λ) ∪ L(R))m for some m ∈ N.
Hence, for each 1 ≤ i ≤ m, ∃ x_i ∈ (L(Λ) ∪ L(R)) such that x = x_1 x_2 x_3 .... x_m.
So in {x_1, x_2, x_3, ...., x_m}, identify those x_1 x_2 x_3 .... x_n where 1 ≤ i_1 < i_2 < .... < i_n ≤ m
And x_{i_j} ∈ L(R) (i.e., x_{i_j} ≠ Λ). Then x = x_1 x_2 x_3 .... x_n or x ∈ L(R)_n ⊆ L(R)*.
Hence, (L(Λ) ∪ L(R))* ⊆ L(R)*.
Therefore, we have (L(Λ) ∪ L(R))* = L(R)* which implies R* = (Λ + R) R*.

6. Construct a DFA for the given regular expression: a*bc* +ac (5 points)
Solution:

7. Write down the transition function for the given NFA: (5 points)
Solution:

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