Homogeneous Coordinates

• Given a 2D point \((x, y)\), its homogeneous coordinates are \((xw, yw, w)\) for any \(w \neq 0\)

\[
(x, y) \rightarrow (xw, yw, w), \quad w \neq 0
\]

• Given a set of homogeneous coordinates \((x, y, w)\), the 2D point it represents is \((x/w, y/w, 1)\)

• Two sets of homogeneous coordinates \((x, y, w)\) and \((x', y', w')\) represent the same 2D points if and only if one is a multiple of the other representations, e.g.

\[
(2, 3, 6) = (4, 6, 12)
\]

• Each point has many different homogeneous coordinate
• Mapping between 2D points and their homogeneous coordinate representations

\[(x^w, y^w, w)\]

• Why homogeneous coordinates?
  - If points are expressed in homogeneous coordinates, all transformations can be treated as vector-matrix multiplications
  - can increase the set of points that can be represented by a computer
Homogeneous Representations of Transformations

- **Rotation:**

\[
\begin{bmatrix}
{x'} \\
y' \\
1
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\cdot
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]

- **Translation**

\[
\begin{bmatrix}
{x'} \\
y' \\
1
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & \Delta x \\
0 & 1 & \Delta Y \\
0 & 0 & 1
\end{bmatrix}
\cdot
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]

- **Scaling**
- **Reflection**
- **Shearing**

**Composition of 2D Transformations**
2D Viewing-transformation pipeline

MC → Construct World-Coordinate Scene Modeling-Coordinate Transformations → WC

WC → Window-To-normalized-Viewport Mapping → NVC

NVC → Normalized-Viewport-To-Viewport Mapping → DC

Clipping against Window
Window-to-Viewport Mapping

\[ x_v = x_{v_{\text{min}}} + (x_w - x_{w_{\text{min}}}) s_x \]

\[ y_v = y_{v_{\text{min}}} + (y_w - y_{w_{\text{min}}}) s_y \]

\( s_x \) and \( s_y \) are scaling factors

\[ s_x = \frac{x_{v_{\text{max}}} - x_{v_{\text{min}}}}{x_{w_{\text{max}}} - x_{w_{\text{min}}}} \]

\[ s_y = \frac{y_{v_{\text{max}}} - y_{v_{\text{min}}}}{y_{w_{\text{max}}} - y_{w_{\text{min}}}} \]