# Tools for modeling and solving search problems

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**Abstract.** In this paper, we describe a language $PS^{pb}$ to model search problems that are specified in terms of *boolean combinations* of pseudo-boolean constraints. We then describe software tools that allow one to use SAT solvers to compute solutions to instances of search problems represented in the language $PS^{pb}$.

## 1 Introduction

Recent research demonstrated that programs computing models of theories in propositional languages, or *SAT solvers*, can be used to find solutions to a broad class of search problems. Due to advances in the performance of SAT solvers, that approach is now becoming practical for an ever expanding range of applications. Despite this computational potential of SAT solvers, the tools to support and facilitate their use are lagging behind. They are *ad hoc* and problem specific. Typically, to use a SAT solver to compute solutions of a search problem $\Pi$, a programmer develops a *specialized* program $P_\Pi$ that generates, for each instance of $\Pi$, a corresponding instance of the SAT problem. That approach makes it difficult to reason about problem constraints as they are "hard-wired" in the program $P_\Pi$. It also hinders the use of SAT solvers as *general-purpose* computational mechanism. Different search problems (even equivalent but different representations of the same problem) require an associated specialized "translator" program.

Our objective is to provide support for a more general and systematic approach to solving search problems by SAT solvers consisting of the following three main steps:

1. **Modeling**: a programmer represents constraints of a search problem $\Pi$ as a theory $P_\Pi$ in a high-level constraint language $\mathcal{L}$, and constructs a description $D_I$ of a specific instance $I$ of the problem $\Pi$.
2. **Compiling**: a specialized program compiles the pair $(D_I, P_\Pi)$ into a theory $T_{\Pi,I}$ in some propositional target logic $\mathcal{L}_{tgt}$ so that solutions to problem $\Pi$ for an instance $I$ correspond to models of $T_{\Pi,I}$ and can be recovered from

them quickly. The compiling program depends only of $\mathcal{L}$ and $\mathcal{L}_{tgt}$ and not on individual search problems.

3. **Solving**: a *SAT solver* for the logic $\mathcal{L}_{tgt}$, that is, a program computing models of theories in $\mathcal{L}_{tgt}$, finds a model of $T_{\Pi,I}$ (and so, also a solution to $\Pi$ for $I$), or determines that no models (solutions) exist.

In this paper we are concerned with the first two phases of that process. Our objective is to design and implement tools that will support the use of *existing* SAT solvers in the third one.

For the modeling language $\mathcal{L}$ in the first step, we design a language to model search problems specified in terms of pseudo-boolean constraints and, more generally, *boolean combinations* of pseudo-boolean constraints. We call this language the language of *propositional schemata with pseudo-boolean constraints* and denote it with $PS^{pb}$. The language $PS^{pb}$ separates the specification of problem constraints from the description of particular data instances. Users model constraints as clauses in the language $PS^{pb}$ and represent problem instances as collections of ground atoms in the language $PS^{pb}$. The language $PS^{pb}$ extends the language $PS+$ [6], which did not allow arbitrary pseudo-boolean constraints.

Syntactically, $PS^{pb}$ clauses are similar to clauses of first-order logic, the only essential difference is a broader class of logic expressions that serve as elementary building blocks (or "atoms"). Semantically, $PS^{pb}$ clauses are interpreted as *propositional schemata* and represent sets of their propositional instantiations (thus, our logic is not a version of first-order logic). Modeling constraints as $PS^{pb}$ clauses makes it possible to automate reasoning about and reformulation of problem specifications. It also makes the language $PS^{pb}$ an effective programming front-end facilitating the use of SAT solvers and SAT(PB) solvers (programs computing models of theories consisting of boolean combinations of pseudo-boolean constraints) as computational engines.

We focus here on this latter aspect. To support finding models of $PS^{pb}$ theories by means of SAT solvers, we introduce the *propositional logic with pseudo-boolean constraints*, $PL^{pb}$, and propose it for the target propositional logic $\mathcal{L}_{tgt}$ for step 2. As we stated, $PS^{pb}$ clauses represent sets of their ground (propositional) instances. By *grounding* a $PS^{pb}$ representation of a search problem $\Pi$ and its instance $I$, we convert it to a theory in the logic $PL^{pb}$. We describe a *grounder* program, *psgrnd*, that automates the task. Our implementation of *psgrnd* extends the scope and improves the performance of an earlier prototype [6]. It is available at `http://www.cs.uky.edu/psgrnd/`.

The logic $PL^{pb}$ extends the logic of pseudo-boolean constraints and, consequently, also the standard propositional logic. If the original $PS^{pb}$ theory contains *no* pseudo-boolean atoms, grounding it with *psgrnd* yields a propositional theory and off-the-shelf SAT solvers can be used in step 3. If the original $PS^{pb}$ theory contains no boolean combinations of pseudo-boolean atoms (each pseudo-boolean atom forms a unit clause), the *psgrnd* program generates a collection of propositional pseudo-boolean constraints and they can be solved by SAT(PB) solvers such as *PBS* [1], *SATZOO* [7] and *SATURN* [11]. Finally, if all pseudo-boolean atoms in the original $PS^{pb}$ theory are *cardinality atoms* (all weights are

equal to 1), then *psgrnd* outputs a propositional theory with cardinality atoms and solvers such as *aspps* [6] and *wsatcc* [10] can be used to compute its models.

To further facilitate the use of existing and future solver programs, we designed a DIMACS-like default output format for *psgrnd* and scripts to translate it, whenever appropriate, into the DIMACS format and to input formats of several SAT(PB) solvers. Our main objective in designing the *psgrnd* format is to establish it as the standard input format for future $PL^{pb}$ solvers, as adopting it will make the use of the $PS^{pb}$ modeling language and the *psgrnd* grounding program even more straightforward and direct.

## 2 Logic $PL^{pb}$

We start with the description of the logic $PL^{pb}$ as it makes it easier later to introduce the logic $PS^{pb}$ as a generalization of $PL^{pb}$ to the first-order language. A *pseudo-boolean atom* (*pb-atom*, for short) is an expression of the form

$$A = l\{p_1 = w_1, \ldots, p_k = w_k\}u,$$

where $p_1, \ldots, p_k$ are atoms from some fixed set $At$, $w_1, \ldots, w_k$ are integer *weights* associated with atoms $p_1, \ldots, p_k$, respectively, and $l$ and $u$, where $l \leq u$, are integers called the *bounds*. One of the bounds, but not both, may be missing. If all weights are equal 1, we often drop them from the notation and write $A$ as $l\{p_1, \ldots, p_k\}u$ (in this case, we refer to these pb-atoms as *cardinality* atoms).

Pb-atoms are just a different notation for *pseudo-boolean constraints* [2], that is, integer-programming constraints $l \leq \sum_{i=1}^{k} p_i w_i \leq u$, where $p_i$s are now regarded as integer variables with the domain $\{0, 1\}$ rather than propositional variables as before. Propositional clauses can be represented as pseudo-boolean constraints [3, 2]. Since pseudo-boolean constraints often make modeling of application problems more direct, the problem of computing assignments satisfying sets of pseudo-boolean constraints has received much attention in the SAT community and resulted in several effective SAT(PB) solvers [2, 13, 1, 11, 7].

Pb-atoms can be combined into more complex constraints. A *pb-clause* is an expression of the form

$$C = \quad A_1, \ldots, A_s \rightarrow B_1 | \ldots | B_t,$$

where all $A_i$ and $B_i$ are (propositional) atoms or pb-atoms[4]. We note that we write ',' and '|' for the conjunction and the disjunction operators, respectively. We also use the "implication" notation for clauses in our approach.

A set of atoms $M \subseteq At$ *satisfies* a pb-atom $A = l\{p_1 = w_1, \ldots, p_k = w_k\}u$, denoted by $M \models A$, if

$$l \leq \sum_{\{i:\ p_i \in M\}} w_i \leq u,$$

[4] Most current SAT(PB) solvers do not accept such complex constraints. They require that each pseudo-boolean constraint represents a "unit" clause. Two exceptions are *aspps* [6] and *wsatcc* [10]. However, they only accept clauses built of cardinality atoms.

with an obvious extension to the case when one of $l$ and $u$ is missing. A set of atoms $M$ *satisfies* a pb-clause $C$, written as $M \models C$, if $M$ satisfies at least one (pb-)atom $B_j$ or does not satisfy at least one (pb-)atom $A_i$.

Since the logic $PL^{pb}$ extends the logic of pseudo-boolean constraints, it also extends the clausal propositional logic. But, there is a more direct relationship. Indeed, every CNF clause

$$\neg a_1 \vee \ldots \vee \neg a_m \vee b_1 \vee \ldots \vee b_n$$

has an equivalent representation in the logic $PL^{pb}$ as an implication (we recall that in the logic $PL^{pb}$ ',' and '|' stand for '$\wedge$' and '$\vee$', respectively)

$$a_1, \ldots, a_m \rightarrow b_1 | \ldots | b_n.$$

In other words, the clausal propositional logic is simply a fragment of the logic $PL^{pb}$.

To illustrate the use of the logic $PL^{pb}$ we will consider the *dominating-set* problem. Let $G = (V, E)$ be an undirected graph. A set $X \subseteq V$ is a *dominating set* in $G$ if every vertex of $G$ is in $X$ or is adjacent to a vertex in $X$. Given an undirected graph $G$, a weight function $w$ assigning integers to vertices, and an integer $k$, the problem is to find a dominating set $X$ such that the total weight of vertices in $X$ is at most $k$. To model the problem, we define a $PL^{pb}$ theory $D(G, w, k)$ as follows:

$\{p_v = w(v) \colon v \in V\}k$
$p_v \mid 1\{p_w \colon \{v, w\} \in E\}$, for every $v \in V$.

It is clear that a set of vertices $X$ is a dominating set with the total weight at most $k$ if and only if the set of atoms $\{p_v \colon v \in X\}$ is a model of $D(G, w, k)$. This example shows one of the advantages of the logic $PL^{pb}$ over formalisms which do not allow boolean combinations of pb-atoms. There is a *direct* mapping of the constraint defining a dominating set to a clause of the logic $PL^{pb}$.

## 3 Language $PS^{pb}$

The main question we deal with in this paper is how to specify theories such as $D(G, w, k)$ so that problem specifications are described concisely by means of finite programs that are independent of particular data instances.

Let us consider the following clauses in the language of predicate logic (to stay consistent with the notation used earlier, we write ',' for '$\wedge$' and '|' for '$\vee$'):

$r(X)|g(X)|b(X)$
$edge(X, Y), r(X), r(Y) \rightarrow \bot$
$edge(X, Y), g(X), g(Y) \rightarrow \bot$
$edge(X, Y), b(X), b(Y) \rightarrow \bot$

These clauses can be viewed as a specification of the graph 3-coloring problem. Indeed, given a set of ground atoms

$$D_G = \{vtx(x)\colon x \in V\} \cup \{edge(x,y)\colon \{x,y\} \in E\},$$

specifying a graph, and some typing information stating that $X$ and $Y$ can only be substituted with constants in the extension of the relation symbol $vtx$, these clauses offer a concise notation for the following collection of their ground instantiations

$r(x)|g(x)|b(x)$, for every $x \in V$
$edge(x,y), r(x), r(y) \rightarrow \bot$, for every $x, y \in V$
$edge(x,y), g(x), g(y) \rightarrow \bot$, for every $x, y \in V$
$edge(x,y), b(x), b(y) \rightarrow \bot$, for every $x, y \in V$.

Under an additional assumption that the truth values of the atoms of the form $vtx(x)$ and $edge(x,y)$ are fully determined by the set $D_G$ (those in $D_G$ are true and those not in $D_G$ are false), these clauses can be rewritten as

$r(x)|g(x)|b(x)$, for every $x \in V$
$r(x), r(y) \rightarrow \bot$, for every $x$ and $y$ such that $edge(x,y) \in D_G$
$g(x), g(y) \rightarrow \bot$, for every $x$ and $y$ such that $edge(x,y) \in D_G$
$b(x), b(y) \rightarrow \bot$, for every $x$ and $y$ such that $edge(x,y) \in D_G$.

This is a version of a familiar propositional encoding of the problem of 3-coloring of the graph $G$. It has the property that there is a one-to-one correspondence between models of that theory and 3-colorings of $G$.

In [6], we described a formalism to model search problems in a way generalizing the graph 3-coloring example. The language of that formalism is essentially a fragment of the standard language of first-order logic with the *signature* $(R_d, R_p, C, V)$, where $R_d$ and $R_p$ are disjoint sets of relation symbols, and $C$ and $V$ are sets of constant and variable symbols, respectively[5]. However, we distinguish two types of relation symbols depending on whether they belong to $R_d$ or $R_p$. We use relation symbols in $R_d$ to represent data instances of search problems and call them *data* relation symbols. We call relation symbols in $R_p$ *program* relation symbols. In the graph-coloring example, $R_d = \{vtx, edge\}$ and $R_p = \{r, b, g\}$.

The definitions of *terms*, *atoms*, and *ground* terms and atoms are standard. A clause in the language is an expression of the form

$$C = \quad A_1, \ldots, A_s \rightarrow B_1| \ldots |B_t,$$

where all $A_i$ and $B_i$ are atoms. As before, we write clauses as implications rather than disjunctions. We recall that we use ',' and '|' in place of '$\wedge$' and '$\vee$'.

[5] The language also contains *predefined* relation symbols ==, <=, <, >= and > for the equality and arithmetic comparisons, and *predefined* function symbols such as +, −, ∗ and / to represent arithmetic operations. For these symbols, we *always* assume their standard interpretation. Consequently, we drop them from the signature.

Given a search problem, we model its particular computational instance by a *data-program* pair $(D, P)$, where $D$ is a set of ground atoms built of data relation symbols and $P$ is a *program*, that is, a set of clauses specifying problem constraints. Programs also contain *typing* declarations. The statements with the keyword *pred* define program relation symbols, specify their arities and the types of the arguments. The statements with the keyword *var* specify types of variables that appear in the program. The types are given by unary data relation symbols. For the graph-coloring example the typing declarations are of the form:

*pred* $r(vtx)$
*pred* $g(vtx)$
*pred* $b(vtx)$
*var* $vtx$ $X, Y$

Let $(D, P)$ be a data-program pair. Typing specifies for each variable its domain (as the extension in $D$ of the data predicate defining its type). That in turn, for every clause in $P$ determines its set of ground instances. Since we assume, as in the graph-coloring example above, that the extensions of data relation symbols are fully specified by the input data instance $D$, we simplify them away from these ground instances. The union of all such ground instances of all clauses in $P$ is a propositional theory, whose models provide the semantics for the data-program pair $(D, P)$.

To sum up, the formalism proposed in [6] is a language for modeling constraints of search problems as programs, that is, sets of declarations and clauses. The data-program pair, consisting of a program and a specification of a particular data instance as a set of ground atoms, represents a propositional theory — a collection of ground instantiations of clauses in the program with respect to constants specified in the data. Models of this propositional theory correspond to problem solutions.

We will now describe the language $PS^{pb}$ that extends the formalism from [6] by means to model *arbitrary* pb-atoms and their combinations. Due to lack of space, our description is rather informal. A grammar providing a precise definition of the syntax is available at `http://www.cs.uky.edu/psgrnd/`.

The signature of the language of the logic $PS^{pb}$ is $(R_d, R_p, C, V, W)$, where $R_d$, $R_p$, $C$ and $V$ are as before, and where $W$ is a set of weight-function symbols. The only terms in the language are arithmetic expressions built of constant and variable symbols in $C \cup V$. We extend, however, the concept of an atom.

A *weighted-set definition* is an expression $S$ of the form $p(t) = w(t')[L] : d_1(s_1) : \ldots : d_m(s_m)$, where $p$ is a program relation symbol, $w$ is a weight-function symbol, $L$ is a list of variables, $d_i$, $1 \leq i \leq m$, are data or predefined relation symbols, and $t$, $t'$ and $s_i$, $1 \leq i \leq m$, are tuples of terms such that all variables appearing in $t'$ appear also in $t$. We call the expression $d_1(s_1) : \ldots : d_m(s_m)$ the *condition* of $S$. Intuitively, $S$ stands for the set of all expressions $p(t) = w(t')$, for which all conditions $d_i(s_i)$, $1 \leq i \leq m$, hold. We note that it is possible for $L$ to be empty. In such case, we omit the list from the notation altogether. It is also possible that $m = 0$. In such case we omit the symbol ':'.

If the weight function is a constant function equal everywhere to 1, we omit it from the notation and write $p(t)[L] : d_1(s_1) : \ldots : d_m(s_m)$.

Each weighted-set definition $S$ is a "template" for sets of weighted atoms. Variables appearing in $S$ that are not in the list $L$ appearing in $S$ are *free*. Grounding them (replacing with constants) yields different instances of the template $S$. Variables that appear in the list $L$ of $S$ are *bound* in $S$. Grounding bound variables in an instance of $S$, yields elements of the set defined by that instance. We formalize these intuitions below, when we formally define the notion of grounding.

A *first-order pb-atom* (or simply pb-atom, if there is no ambiguity) is any expression

$$l\{S_1; S_2; \ldots; S_k\}u,$$

where $l$ and $u$ are terms and $S_1, S_2, \ldots, S_k$ are *weighted-set definitions*. Intuitively, the meaning of a predicate pb-atom $l\{S_1; S_2; \ldots; S_k\}u$ is that the total weight of all atoms that are true in the union of the sets specified by the set definitions $S_1, \ldots, S_k$ is at least $l$ and no more than $u$ (we will shortly make this intuition precise). A *pb-clause* is an expression of the form

$$C = \quad A_1, \ldots, A_s \rightarrow B_1 | \ldots | B_t,$$

where all $A_i$ and $B_i$ are atoms or pb-atoms.

We represent an instance of a search problem as a set $D$ of ground atoms specifying the extensions of data relation symbols in $R_d$. The set $D$ also contains expressions of the form $w(c_1, \ldots, c_k) = u$, where $w$ is a $k$-ary weight-function symbol in $W$, $c_1, \ldots, c_k$ are constants that occur in ground atoms listed in $D$, and $u$ is an integer. These expressions define a weight function $w$ (we assume that argument tuples not listed explicitly do not belong to the domain of $w$). We call such sets $D$ *data sets*.

We represent the constraints specifying the search problem itself by a collection of pb-clauses, and typing and declaration statements. The latter have the same format as that we introduced earlier when discussing the graph-coloring example. We call such collections *programs*.

We call a pair $(D, P)$, where $D$ is a data set and $P$ is a program, a *data-program pair*. We will now define *models* of data-program pairs. The definition is based on the interpretation of clauses as *propositional schemata*, that is, as shorthands for sets of *ground* propositional clauses.

Let $(D, P)$ be a data-program pair. First, we consider a weighted-set definition $S$ of the form $p(t) = w(t')[L] : d_1(s_1) : \ldots : d_m(s_m)$ appearing in a pb-clause of a data-program pair $(D, P)$. Let $\vartheta$ be a ground substitution whose domain contains all free variables in $S$ and does not contain any variables that are bound in $S$ (we note that the sets of free and bound variables are disjoint). By $S\vartheta$ we denote the set of expressions of the form $p(t\vartheta\vartheta') = v$, where

1. $\vartheta'$ is a ground substitution with the domain consisting of all variables that are bound in $S$ such that for every $i$, $1 \leq i \leq m$, $d_i(s_i\vartheta\vartheta')$ holds (we note that term tuples $s_i\vartheta\vartheta'$ are ground and, since data relation symbols are fully

specified by a data-program pair, this latter condition can be verified efficiently)
2. $v$ is an integer to which the weight expression $w(t'\vartheta\vartheta')$ evaluates. That is, we evaluate the term tuple $t'\vartheta\vartheta'$, which is ground (performing arithmetic operations, if necessary) and look up in $D$ the value of the function $w$ for the resulting tuple of the ground arguments (we recall that weight functions are fully defined in the data component $D$). The whole expression is undefined if $w$ is undefined for the term $t'\vartheta\vartheta'$.

To specify the meaning of a pb-clause $C$ occurring in the program $P$ of the data-program pair $(D, P)$, we ground $C$ and replace it with a set of propositional pb-clauses. Let us consider a pb-atom $A = l\{S_1; \ldots; S_k\}u$ appearing in $C$. We start by renaming all bound variables by new unique names different from any other variable name in the clause (the renaming does not change the meaning of any of the set definitions in $A$). In this way, the sets of bound and free variables in $C$ are disjoint. Let $\vartheta$ be a ground substitution whose domain contains all free variables and none of the bound ones. We define $A\vartheta$ as follows:

1. $A\vartheta = \bot$, if $l\vartheta$ or $u\vartheta$ are not integers
2. $A\vartheta = \bot$, if for some $1 \leq i \leq k$, $S_i\vartheta$ is undefined
3. $A\vartheta = \bot$ if for some $1 \leq i < j \leq k$ and $w \neq w'$, there are expressions $a = w$ and $a = w'$, in $S_i\vartheta$ and $S_j\vartheta$, respectively.
4. $A\vartheta = l\vartheta\{S_1\vartheta \cup \ldots \cup S_k\vartheta\}u\vartheta$, otherwise. In this case, $l\vartheta$ and $u\vartheta$ are integer constants, and $S_1\vartheta \cup \ldots \cup S_k\vartheta$ is a set of ground expressions of the form $a = w$.

It is clear that $A\vartheta$ is a propositional pb-atom. Applying $\vartheta$ to all atoms in $C$ produces a propositional pb-clause $C\vartheta$. We now define $grnd(D, P)$ to consist of all propositional pb-clauses of the form $C\vartheta$, where $C$ is a pb-clause in $P$ and $\vartheta$ is a ground substitution that contains in its domain all free variables in $C$ and none of $C$'s bound variables. We define a set $M$ of ground atoms to be a *model* of $(D, P)$ if it is a model of $grnd(D, P)$.

Given a mapping assigning to an instance $I$ of a search problem $\Pi$ a data set $D_I$, we say that a program $P$ *solves* $\Pi$ if for every instance $I$, solutions to $\Pi$ for $I$ correspond to models of the data-program pair $(D_I, P)$.

## 4 *Psgrnd*

Models of a data-program pair $(D, P)$ are models of a $PL^{pb}$ theory $grnd(D, P)$. Consequently, they can be computed by SAT solvers for the logic $PL^{pb}$. To facilitate use of such solvers in computing solutions to search problems represented in the language $PS^{pb}$ as data-program pairs, we implemented a program *psgrnd*. Given a data-program pair $(D, P)$, *psgrnd* outputs its grounding $grnd(D, P)$[6].

[6] There are some minor differences between the grounding as we described it and what we implemented in *psgrnd*. Namely, in the cases (1) - (3) of the definition of $A\vartheta$, the grounder produces an error message and terminates with failure.

This section describes the program *psgrnd*. Our implementation is a major enhancement of a prototype program described in [6]. It is based on a formal grammar for the language of the logic $PS^{pb}$. Both the grammar and the *psgrnd* program are available at `http://www.cs.uky.edu/psgrnd/`. To generate the parser source code, we processed this grammar by the Bison utility [9]. We wrote the code of *psgrnd* in C++ and compiled it both under UNIX and Windows environment. We used gcc 3.3 compiler for UNIX and Microsoft Visual Studio 6.0 compiler for Windows XP. The main improvements with respect to the earlier version of the grounder program include the capability to process weight constraints and an option to execute the complete single-atom lookahead to reduce the size of the propositional theory produced by the grounder.

The output from the grounder program when executed on a data-program pair $(D, P)$ is a set of ground instantiations of pb-clauses in $P$ computed with respect to data specification in $D$. It also includes the set of atoms whose logical value was determined by the grounder and those that the grounder determined to be irrelevant (they may assume any logical value in any model of the pair $(D, P)$). Some of the most important options of the program include output in human-readable form, output in the DIMACS format (for input data-program pairs without pb-atoms), disabling lookahead, and disabling propagation.

This implementation of the grounder is often much faster than the previous implementation due to carefully designed memory management module. Moreover, information about data predicates is reorganized before parsing clauses by replacing linked lists of possible values with sorted arrays. This allows us to use binary search during parsing and grounding clauses. The benefits of using binary search exceed additional cost associated with sorting data.

Furthermore, when generating ground instances of clauses of $PS^{pb}$ programs, we ground variables in the order of minimal cost. This eliminates early those variable instantiations which do not lead to valid ground clauses.

Another improvement in efficiency comes from storing names and other multi-character symbols appearing in user input in a symbol table and representing every name in data structures by its corresponding number. As a result, we can often replace inefficient string comparisons with comparisons of two integers.

Previous versions of the program used a binary search tree to store names of ground atoms during grounding process. The current version uses a balanced binary tree data structure for this purpose. In many applications, the number of ground atoms is large (measured in millions). We believe that the use of a balanced binary tree is needed to maintain the set of ground atoms and to search it efficiently.

During the grounding process, when a new clause, say $r$, is added to the collection of ground clauses and $r$ contains only one atom, say $A$, *psgrnd* triggers *unit propagation*. It follows a general format of unit propagation for propositional CNF theories, modified to handle the case of pb-atoms. When there are no more truth values to propagate and no contradictory pb-clause (empty antecedent and empty consequent) was derived, the unit propagation process terminates. If a

contradictory clause was derived, the theory is inconsistent, the whole grounding process terminates and a single contradictory clause is returned.

When grounding is complete, *psgrnd* has an option to perform a complete one-atom lookahead. For every ground atom with an undetermined truth value, the grounder assigns to this atom a truth value (first true, then false) and tentatively propagates it. If this tentative propagation results in an inconsistency, the atom must have the opposite truth value and permanent propagation is later performed on this atom with its assigned value. If a tentative propagation terminates without a conflict, the program restores the theory to its previous state. A complete one-atom lookahead is costly (runs in time $O(n^2)$, where $n$ is the number of ground atoms) but sometimes results in a much smaller theory.

We will now briefly discuss the performance of *psgrnd*. The key question is how fast it can process data-program pairs leading to large ground theories with millions of atoms and clauses. We tested *psgrnd* on data-program pairs $(D_n, P)$, where $P$ is a program consisting of two simple clauses, each with four variables over the same domain, and $D_n$ specifies this single domain as the integer range $(1..n)$. The ground theory for a data-program pair $(D_n, P)$ consists of $2n^4$ ground atoms and $2n^4$ ground clauses. Thus, even for small values of $n$, the ground theory is large, which poses a challenge for grounding programs (for instance, $grnd(D_{50}, P)$ has over 12 million atoms and 12 million clauses). We compared *psgrnd* with its earlier and restricted version [6], which we call here *old-psgrnd*. We also compared *psgrnd* with *lparse*, a state-of-the-art program for grounding DATALOG$^\neg$ programs (for that comparison, we replaced the program $P$ with an equivalent DATALOG$^\neg$ program of a similar structure to $P$). The results for $n = 10$, 20, 30, 40 and 50 are shown in Table 1.

**Table 1.** Hand-made program, large ground theories, CPU time in seconds

| grounder | $n = 10$ | $n = 20$ | $n = 30$ | $n = 40$ | $n = 50$ |
|---|---|---|---|---|---|
| *psgrnd* | 0.09 | 1.8 | 10.59 | 36.43 | 94.76 |
| *old-psgrnd* | 23.68 | 509.30 | 2995.87 | - | - |
| *lparse* | 0.15 | 2.51 | 12.87 | 40.48 | 102.88 |

The results indicate that *psgrnd* is at least two orders of magnitude faster then its earlier version, which timed out (the limit was set at 3600 sec of CPU time). It is also slightly faster than *lparse*. We conducted these experiments on a machine with 3.2GHz Intel Pentium processor, 1Gb memory and running Slackware 9.0 Linux kernel 2.4.25.

We also experimented with programs encoding more typical search problems. For lack of space, we only provide results for the graph coloring problem for certain simplex graphs with $5n$ vertices and $11n - 4$ edges, for $n = 5000$, 10000, 15000 and 20000 (the ground theory in this last case has 300000 atoms and 1059998 clauses). The results in Table 2 show that *psgrnd* significantly outper-

forms *old-psgrnd* and is faster than *lparse* (the latter program causes segmentation fault when run for $n = 20000$).

**Table 2.** Graph-coloring problem

| grounder | $n = 5000$ | $n = 10000$ | $n = 15000$ | n=20000 |
|---|---|---|---|---|
| *psgrnd* | 1.14 | 2.28 | 3.43 | 4.51 |
| *old-psgrnd* | >1000 | >1000 | >1000 | >1000 |
| *lparse* | 1.84 | 3.74 | 5.70 | seg fault |

The program *psgrnd* is invoked from a UNIX command line in the following way

$$psgrnd\ [-d\ dataFileList]\ -r\ ruleFile\ [-c\ constantList]\ [output]\ [flags]$$

where optional *dataFileList* is one or more data files

$$dataFile1\ dataFile2\ \ldots\ dataFileN$$

and *output* is of the form

$$[-o\ outputForSolver]\ [-m\ humanReadableOutput]$$

If *-o* option is not specified, the default output file is *out.aspps*. An optional *constantList* is a list of value assignments for symbolic constants which have to be provided for a particular problem encoding

$$name1 = value1\ \ name2 = value2\ \ldots\ nameM = valueM$$

Flags $-d$, $-r$ , and $-c$ may be dropped, but in this case they should be dropped altogether and the data files should always precede the rule file. *Psgrnd* has several options specifying output formats and processing instructions. We refer to `http://www.cs.uky.edu/psgrnd/` for details.

The program *psgrnd* can currently produce its output in several formats. The default format is designed for the case of *general* theories in the logic $PL^{pb}$. It is accepted by solvers *aspps* [6] and *wsatcc* [10] (if all pb-atoms in the ground theory are cardinality atoms).

To support the use of existing SAT solvers and SAT(PB) solvers, we developed simple scripts translating the *psgrnd* format into DIMACS format (if the ground theory contains no pb-atoms) and into input formats of SAT(PB) solvers *PBS* [1], *SATZOO* [7] and *SATURN* [11] (if all clauses containing pb-atoms are unit clauses). The script generating DIMACS format is already integrated as an option of *psgrnd*. The other scripts will be integrated in the near future.

We will now briefly describe the default output format for *psgrnd*. Our objectives are to establish that format as the standard input format for solvers for

the logic $PL^{pb}$ and to support the use of $PS^{pb}$ as a programming front-end for existing and future SAT(PB) solvers.

The default output format for *psgrnd* has the following properties. The output file starts with a header line

$$p \;\; number\_of\_propositional\_atoms \;\; number\_of\_pb\_atoms \;\; number\_of\_rules$$

Next $number\_of\_rules$ lines represent clauses of the ground program. Propositional atoms are represented by positive integers. Pseudo-boolean atoms are represented in the form

$$[\; l \; u \; a_1 \; = w_1 \; \ldots \; a_k \; = w_k \;]$$

where $l$ and $u$ are integers representing the lower and the upper bound, respectively, $a_1, \ldots, a_k$ are integers representing propositional atoms, and $w_1, \ldots, w_k$ are integer weights assigned to these atoms. Cardinality atoms (special case of pseudo-boolean atoms where all weights are equal to 1) are represented as expressions

$$\{\; l \; u \; a_1 \; \ldots a_k \;\}$$

with the meaning of $l$, $u$, and $a_1, \ldots, a_k$ as above. Clauses of the ground program contain first (possibly empty) set of atoms in the body, next a comma and, finally, a (possibly empty) set of atoms in the head. Within sets of atoms in the body and in the head, propositional atoms are followed by cardinality atoms, and then by other pb-atoms. After lines representing clauses, there are lines containing description of propositional atoms in the form

$$c \; atom\_number \; atom\_name$$

Atoms appearing in the rules are assigned positive numbers starting with 1. Atoms determined during grounding to be true are assigned number 0. Atoms that the grounder determined to be irrelevant (they may assume any logical value in any model) are assigned number -1. In general, if a line starts with a '$c$' character, it is considered a comment, not a part of the ground program.

## 5 Examples

We will now consider several search problems and describe $PS^{pb}$ programs that solve them. All relevant data relation symbols and weight function symbols appear in typing statements and in clauses of programs solving search problems. Therefore, we will only describe programs and omit a detailed discussion of the data representation schemata.

We start with the dominating set problem, which we discussed earlier. The program solving the problem consists of the following statements and clauses:

```
pred in(vtx).
var vtx X,Y.

{in(X)=w(X)[X]}k.
in(X) | 1{in(Y)[Y]: edge(X,Y)}.
```

The first clause captures the constraint that the sum of weights of selected vertices is at most $k$. The second clause represents the constraint defining a dominating set: every vertex belongs to the set or at least one of its neighbors does[7]. This program (given a data set) grounds to the $PL^{pb}$ theory we described in Section 2. We note that following the grammar of the syntax of the language $PS^{pb}$ accepted by *psgrnd*, we complete each declaration and each clause with a period '.'. We also note that the constant $k$ appearing in the first clause and specifying the bound on the total weight of a dominating set needs to be specified at the command line when calling *psgrnd*.

The next problem we discuss is the $n \times n$ magic square problem. To solve it we can use the following $PS^{pb}$ program.

```
pred in(index,index,entry).
var index I,J.
var entry K.

1 {in(I,J,K)[K]} 1.
1 {in(I,J,K)[I,J]} 1.
n*(n*n+1)/2 {in(I,J,K)=w(K)[J]} n*(n*n+1)/2.
n*(n*n+1)/2 {in(I,J,K)=w(K)[I]} n*(n*n+1)/2.
n*(n*n+1)/2 {in(I,I,K)=w(K)[I]} n*(n*n+1)/2.
n*(n*n+1)/2 {in(I,n+1-I,K)=w(K)[I]} n*(n*n+1)/2.
```

The first two clauses are unit cardinality atoms that ensure that (1) there is exactly one value $K$ (from the range $(1..n^2)$ defined in the data set) for every position $(I, J)$ in the array, and that (2) every value $K$ is placed in some position $(I, J)$. The remaining four clauses describe the basic problem constraints that each row, column and two main diagonals have entries that sum up to the same value $n(n^2+1)/2$. As before, the constant $n$ needs to be specified at the command line, when calling *psgrnd*.

Finally, we consider the Schur problem: given integers $k$ and $n$, find an assignment of $1, \ldots, n$ into $k$ bins so that each bin is sum-free (if $x$ and $y$ are in a bin, $x + y$ is not). We can solve that problem with the following program.

```
pred in(num,part).

var num M,N.
var bin B.

1{in(M,B)[B]}.
in(M,B), in(N,B), in(M+N,B) -> .
```

The first clause captures the requirement that each integer $M$ (in the range specified in the data set), is assigned to some bin $B$. The second clause describes the Schur constraint.

[7] We assume here that every edge $\{x, y\}$ of an input graph is represented both as $edge(x, y)$ and $edge(y, x)$.

Data-program pairs with these three programs ground to theories in the logic $PL^{pb}$. In the last two cases, these theories consist of pseudo-boolean constraints and SAT(PB) solvers can be used to compute their models (solutions to the corresponding instances of the search problems). The full syntax of $PS^{pb}$, not discussed here due to the lack of space, allows us to write the first clause as

```
in(M,B)[B].
```

The meaning of this clause is similar to the one used before. The difference is that it grounds to a collection of *propositional clauses* of the form $in(m,1)|\ldots|in(m,k)$ rather than to propositional pb-atoms of the form $1\{in(m,1),\ldots,in(m,k)\}$. Clearly, both types of ground expressions have the same semantics. However, now the corresponding data-program pairs ground to propositional CNF theories and *standard* SAT solvers can be used for computing solutions.

## 6 Conclusions, related and future work

We defined a language $PS^{pb}$ for modeling search problems specified by *boolean combinations* of pseudo-boolean constraints. Based on a formal grammar of $PS^{pb}$, we designed and implemented a program *psgrnd* that converts specifications of search problems in the language $PS^{pb}$ into theories in the logic $PL^{pb}$. The theories generated by *psgrnd* can be output in formats accepted by current SAT solvers for CNF theories and by solvers for more general $PL^{pb}$ theories (at present, under some syntactic restrictions). In this way, the *psgrnd* program facilitates the use of SAT solvers as a general computational mechanism for finding solutions to search problems modeled in the language $PS^{pb}$.

Other researchers also studied the problem of modeling propositional and pseudo-boolean constraints. The closest to our work are the language *ESO* (the existential fragment of the second-order logic) [5, 4] and the language *QPROP*, extending the language of propositional logic with finite quantification [8]. Each of these languages is a restricted version of our language in that they do not admit pseudo-boolean constraints.

This paper represents work in progress. In the near future, we will extend the language $PS^{pb}$ with optimization statements, and will allow negative integers in pb-atoms. We will also provide concise ways to define weight functions in terms of arithmetic expressions.

As we mentioned at the beginning, representing search problems in the language $PS^{pb}$ makes it possible to reason about them. Two important problems stemming from that possibility are: (1) to develop automated techniques to rewrite problem specifications so that to improve the performance of SAT solvers on the corresponding ground theories produced by *psgrnd*, and (2) to design a class of SAT solvers whose heuristics could be customized to a particular ground theory based on properties of its high level specification in the language $PS^{pb}$.

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