Homework 7: CS537, Spring 2017
Due Date: 10:50am, April 21, 2017

Please show all steps in your work. Please be reminded that you should do your homework independently.

1. (10 points) Consider the problem $x' = x$. If the initial condition is $x(0) = c$, then the solution is $x(t) = ce^t$. If a roundoff error of $\epsilon$ occurs in reading the value of $c$ into the computer, what effect is there on the solution at point $t = 10$? At $t = 20$? Do the same for $x' = -x$.

2. (10 points) Find a polynomial $p$ with the property $p - p' = t^3 + t^2 - 2t$.

3. (10 points) The function $f(x, y) = xe^y$ can be approximated by the Taylor series in two variables by $f(x + h, y + k) \approx (Ax + B)e^y$. Determine $A$ and $B$ when terms through the second partial derivatives are used in the series.

4. (10 points) Solve the differential equation

$$\begin{cases} \frac{dx}{dt} = -tx^2 \\ x(0) = 2 \end{cases}$$

at $t = -0.2$, correct to two decimal places, using one step of the Taylor series method of order 2 and one step of the Runge-Kutta method of order 2.

5. (10 points) Solve the differential equation $x' = x$ with initial value $x(0) = 1$ by the Taylor series method on the interval $[0, 10]$. Compare the result with the exact solution $x(t) = e^t$. Use derivatives up to and including the tenth. Use step size $h = 1/100$.

6. (10 points) Consider the third-order Runge-Kutta method:

$$x(t + h) = x(t) + \frac{1}{9}(2K_1 + 3K_2 + 4K_3)$$

where

$$\begin{cases} K_1 = hf(t, x) \\ K_2 = hf(t + \frac{1}{2}h, x + \frac{1}{2}K_1) \\ K_3 = hf(t + \frac{1}{2}h, x + \frac{1}{2}K_2) \end{cases}$$

Show that it agrees with the Taylor series method of the same order for the differential equation $x' = x + t$. 