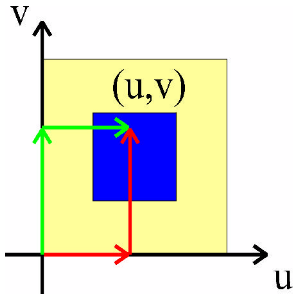
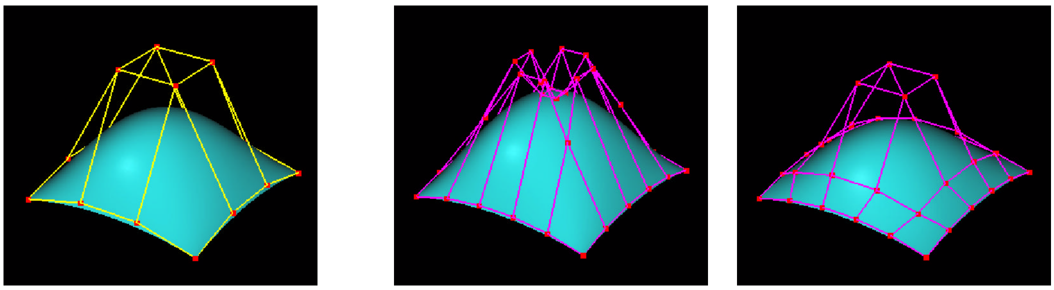


Figure 10. Advantage of the two-path approach.

3.3 Boundary Constraints for Surfaces

There are many occasions in the design process that one needs to hold the boundary curves of a surface unchanged while manipulating or deforming the surface. Fixed boundary implies that integration along the boundary curves is not required in calculating $\boldsymbol{T}(u,v)$, only the integrals along an internal path are needed.

Once the control points of the surfaces $\boldsymbol{T}_{uv}(u,v)$ and $\boldsymbol{T}_{vu}(u,v)$ defined by Eq.(8) have been computed, the boundary condition is achieved by applying the end-point constraint for the curve case to each row (column) of control points of $\boldsymbol{T}_{uv}(u,v)$ and $\boldsymbol{T}_{vu}(u,v)$ in v and u directions, respectively. Consider, for example, $\boldsymbol{T}_{uv}(u,v)$. Since the internal path is in v direction, each column of control points of $\boldsymbol{T}_{uv}(u,v)$ in v direction is relocated by a rotation and a scaling to enforce the end-point constraint of that column. The same operation is performed on rows of control points of $\boldsymbol{T}_{vu}(u,v)$ in u direction. Figure 11 shows examples of fine-tuned surfaces with the end-point(boundary) constraint. The original surface is a bi-cubic B-spline surafce and a bi-quadratic B-spline surface with 3×3 control values is used as the scalar function and the surface is fine tuned by decreasing and increasing only the center control value from 1 to -1.7 and to 16, respectively, using the two-path integration.



(a) Original surface.

(b) Fine-tuned (central control value $= -1.7$).

(c) Fine-tuned surface (central control value $= 16$).

Figure 11. Examples of surface fine tuning. (a) Original surface. (b): Fine-tuned surface (central control value $= -1.7$). (c) Fine-tuned surface (central control value $= 16$).

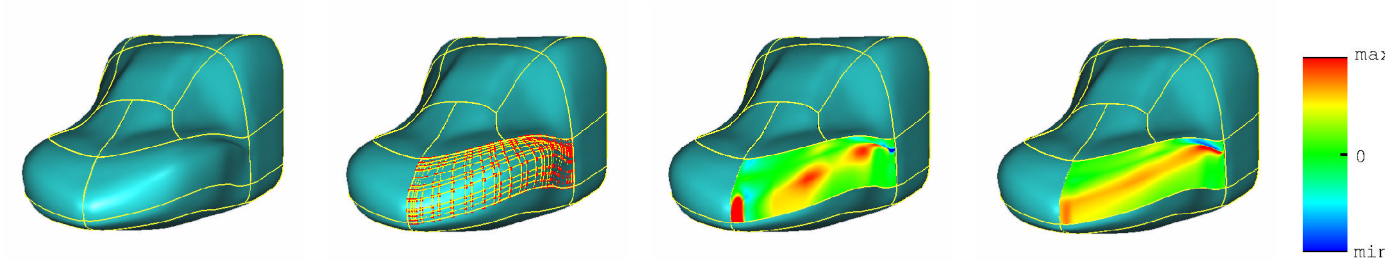
Tangent and curvature continuity across the boundary curves can be accomplished in the same manner as the curve case. For example, to maintain tangent continuity across the boundary curves, the second lows (columns) of the control points from the shared boundaries of the adjacent surfaces have to be fixed and matched with those of the fine-tuned surface. This is done by multiplying the derivative of the fine-tuned surface with an identity scalar function which is equal to 1 everywhere, and then by applying $\alpha(u,v)$ and by performing translation, rotation and scaling. For curvature, the third lows (columns) of the control points should be matched.

Because of the same reason as the curve case, the above method does not work for closed surfaces. Like the curve case, closed surfaces need different treatment which will be discussed in Section 4.

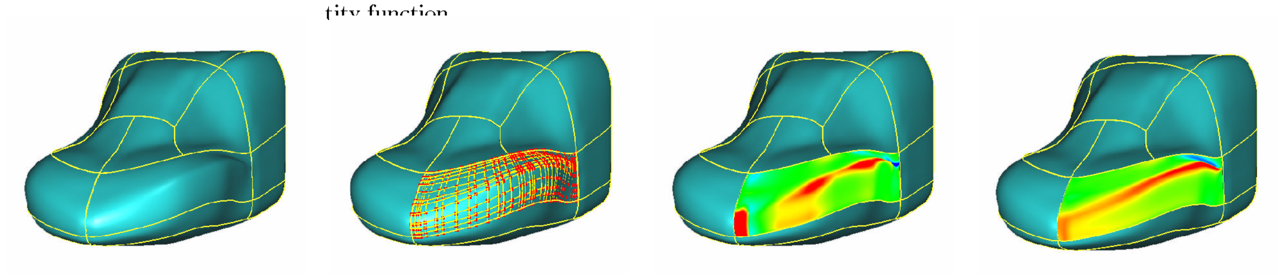
Figure 12 shows an example of fine-tuned surfaces with the curvature constraint. A bi-quadratic B-spline surface with 3×4 control values is used as the scalar function $\alpha(u,v)$. All the control values of the scalar function are set to 1 except the two central control ones which are set to -0.8 in this example.[2] Since the surface to be fine tuned

[2]The control value itself is negative, but the value of the scalar function is always positive and the direction of the derivatives are not reversed.

is much longer in one parameter direction, the one-path integration approach in the other (shorter) parameter direction is adopted for the fine tuning process. Since the small values are specified for the central control values, the middle part of the surface is sharpened along the longer parameter direction. The basic shape of curvature distribution is kept as shown in (c), (d), (h), and (i).



(a) Original surface(bi-cubic B-spline surface). (b) Control points of fine-tuned surface with an identity function. (c) Curvature distribution of the original (Gaussian). (d) Curvature distribution of the original (mean). (e) Color.



(f) Fine-tuned surface(bi-quintic B-spline surface). (g) Control points of fine-tuned surface. (h) Curvature distribution of the fine-tuned surface (Gaussian). (i) Curvature distribution of the fined-tuned surface (mean).

Figure 12. Example of surface fine tuning. (a) Original surface. (b) The control points of the original surfaces. (c) and (d) are Gaussian and mean curvature distributions of the original. (e) Color assignment for curvature display. (f) Fine-tuned surface. (g) The control points of the fine-tuned surface. (h) and (i) are Gaussian and mean curvature distributions of the fined-tuned surface.

3.4 Local Fine Tuning of a Surface

With the capability of holding the boundary curves unchanged during the fine tuning process and maintaining tangent and curvature continuity across the boundary curves of a surface patch, one can actually perform fine tuning on any rectangular portion of a parametric surface. An example is shown in Figure 13. A bi-quadratic B-spline surface with 3×3 control values is used as the scalar function and the surface is locally flatted by increasing only the center control value from 1 to 4 using the one-path integration.

3.5 Processing Time

Table 1 summarizes the processing times of the test cases shown in the paper. The platform is a Pentium III 1GHz computer.[3] For each test case, we show the degree and number of control points of the original shape, the degree and number of control values of the scalar function, and the processing time. The processing time covers all the operations performed, i.e., differentiation, product, and integration.

In the case of curve fine tuning, as can be seen from the table, the processing times of even rather complicated shapes are small enough for an interactive environment. The processing times of surface fine tuning, on the other hand, can not completely achieve interactive effect yet. Our current implementation is based on the recurrence relation of the B-splines [9]. A significant speedup can be expected by tabulating the product formulas with

[3] All the processing times in the paper are measured on a Pentium III 1 GHz machine.